Math 54-1 Your name: Key
Quiz 8, July 23, 2010

Please write your name **on each sheet**. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (5 pt) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}.$$

For each eigenvalue, state its algebraic multiplicity and find a basis for the corresponding eigenspace.

A is upper triangular → the characteristic polynomial is $(-1-\lambda)\cdot(-\lambda)(-1-\lambda) = (-1-\lambda)^2(0-\lambda)$ → eigenvalues are 0 (mult. 1) and -1 (mult. 2)

$\underline{\lambda = 0}$ → Eigenspace is $\text{Nul } A = \text{Nul}\begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} =$

$= \text{Nul}\begin{bmatrix} \boxed{1} & -1 & 0 \\ 0 & 0 & \boxed{1} \\ 0 & 0 & 0 \end{bmatrix}$; basis $\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right\}$

$\underline{\lambda = -1}$ → Eigenspace is $\text{Nul}(A - \lambda I) = \text{Nul}(A + I) =$

$= \text{Nul}\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \text{Nul}\begin{bmatrix} 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; basis $\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right\}$.

2. (5 pt) Compute the characteristic polynomial and find the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}.$$

State the algebraic multiplicity of each eigenvalue.

$$p(\lambda) = \det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 1 \\ -1 & 4-\lambda \end{vmatrix} =$$

$$= (\lambda-2)(\lambda-4) + 1 = \lambda^2 - 6\lambda + 9 = (\lambda-3)^2$$

The only eigenvalue is $\lambda = 3$, multiplicity 2.