Math 54-1
Quiz 6, July 16, 2010

Your name: Key

Please write your name **on each sheet.** Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (5 pt) Find the coordinate vector of

$$\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

in the following basis of $\mathbb{R}^2$:

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}.$$

We need to solve the vector equation $\begin{bmatrix} 1 \\ 0 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Augmented matrix:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -2 \end{bmatrix} \longrightarrow$$

$$\xrightarrow{R_2 \leftarrow \frac{-R_2}{3}} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2/3 \end{bmatrix} \xrightarrow{R_1 \leftarrow R_1 - 2R_2} \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 2/3 \end{bmatrix}.$$

So, $\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = [\vec{v}]_{\mathcal{B}} = \begin{bmatrix} -1/3 \\ 2/3 \end{bmatrix}$.

2. (5 pt) Use coordinate vectors to decide whether the polynomials $t, (1-t)^2, (1+t)^2$ are linearly independent.

Consider the basis $\{1, t, t^2\}$ of $\mathbb{P}_2$. The coordinate vectors wrt this basis are:

$$[t] = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad [(1-t)^2] = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \quad [(1+t)^2] = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Now, $\{t, (1-t)^2, (1+t)^2\}$ are Lin. Ind. $\iff$

$\iff \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ are Lin. Ind. $\Leftarrow$

$\Leftarrow \begin{bmatrix} 0 & 1 & 1 \\ 1 & -2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ has a pivot in each column.

Row reduce: $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, no pivot in column 3.

So, $\{t, (1-t)^2, (1+t)^2\}$ are linearly dependent.

(In fact, $4t = (1+t)^2 - (1-t)^2$.)