Your name: ___Key___

Please write your name **on each sheet**. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (4 pt) Find

$$\det \begin{bmatrix} 2 & 2 & 0 \\ 3 & 3 & 3 \\ 0 & 1 & 1 \end{bmatrix}.$$

$$\det \begin{bmatrix} 2 & 2 & 0 \\ 3 & 3 & 3 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow[\substack{R_1 \leftarrow R_1/2 \\ R_2 \leftarrow R_2/3}]{} 2 \cdot 3 \cdot \det \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} =$$

$$\xrightarrow{R_2 \leftarrow R_2 - R_1} 6 \det \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = 6 \det \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = -6.$$

2. (6 pt) Let
$$A = \begin{bmatrix} t & 1 \\ -1 & -t \end{bmatrix}.$$

(a) Find all values of t for which the matrix A is invertible.

(b) For t such that A is invertible, find A^{-1} and use it to solve the equation
$$A\vec{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

(a) A invertible $\iff$ det $A \neq 0$ $\iff$

$\iff$ $1 - t^2 \neq 0$ $\iff$ $t \notin \{1, -1\}$.

(b) Assume that $t \neq \pm 1$. Then

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} -t & -1 \\ 1 & t \end{bmatrix} = \frac{1}{1-t^2} \begin{bmatrix} -t & -1 \\ 1 & t \end{bmatrix}$$

$$\vec{x} = A^{-1} \begin{bmatrix} 1 \\ -1 \end{bmatrix} =$$

$$A\vec{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \iff$$

$$= \frac{1}{1-t^2} \begin{bmatrix} -t & -1 \\ 1 & t \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{1-t^2} \begin{bmatrix} 1-t \\ 1-t \end{bmatrix} = \frac{1}{1+t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$