Math 54-1 Your name: Key

Quiz 3, July 6, 2010

Please write your name **on each sheet**. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (3 pt) ~~(a)~~ Assuming that the transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by the formula

$$T(x_1, x_2, x_3) = (x_1 - x_2, x_2 - x_3)$$

is linear, find its standard matrix.

Bonus (no points on this quiz, but might come up on the midterm): Is T 1-to-1? Is is onto?

The standard matrix is $\left(\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \vec{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right)$

$$A = \begin{bmatrix} T(\vec{e}_1) & T(\vec{e}_2) & T(\vec{e}_3) \end{bmatrix} =$$

$$= \begin{bmatrix} T(1,0,0) & T(0,1,0) & T(0,0,1) \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}.$$

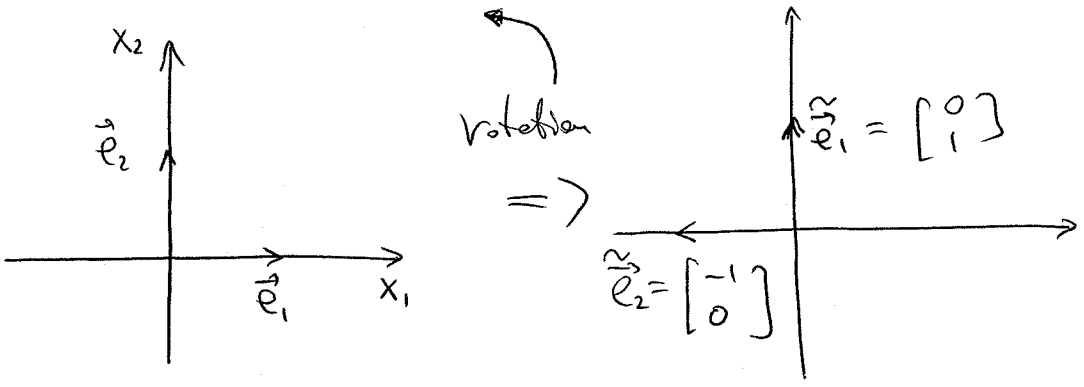
Bonus: Row reduce $A \to$ already in REF $\begin{bmatrix} \boxed{1} & -1 & 0 \\ 0 & \boxed{1} & -1 \end{bmatrix}$

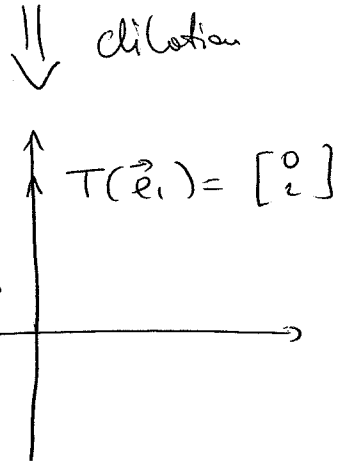
Pivot in each row $\to$ A is onto

No pivot in column 3 $\to$ A is not 1-to-1.

2. (4 pt) Write the standard matrix for the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that first rotates points $\pi/2$ radians counterclockwise and then dilates points by 2 (that is, multiplies each vector by 2). You can use matrix multiplication if you want, but you have to explain your steps.

Put $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Then





Standard matrix: $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

Alternatively, Standard matrix of rotation: $\begin{bmatrix} \cos \pi/2 & -\sin \pi/2 \\ \sin \pi/2 & \cos \pi/2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Standard matrix of dilation: $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

Then, $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$.

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3. (3 pt) Find A^TA, where

$$A = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}.$$

$$A^T = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix},$$

$$A^TA = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$