Math 54-1 Your name: Key
Quiz 14, August 12, 2010

Please write your name **on each sheet**. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (6 pt) Find the Fourier sine series of the function

$$f(x) = x,\ 0 < x < \pi,$$

and describe and sketch the graph of the function to which this series converges on the whole real line.

We have $f(x) \sim \sum_{k=1}^{\infty} b_k \sin(kx)$, where

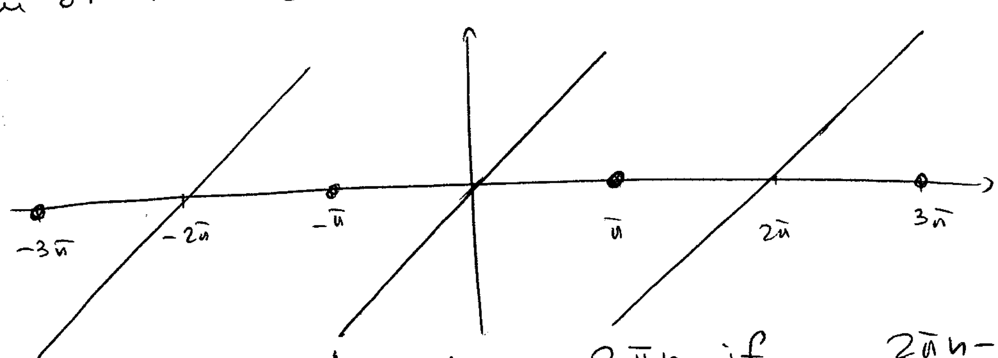
$$b_k = \frac{2}{\pi}\int_0^{\pi} x\sin(kx)\,dx = -\frac{2}{\pi k}\int_0^{\pi} x\,d(\cos(kx)) =$$

$$= -\frac{2}{\pi k} x\cos(kx)\Big|_{x=0}^{\pi} + \frac{2}{\pi k}\int_0^{\pi}\cos(kx)\,dx =$$

$$= -\frac{2}{k}\cos(\pi k) + \frac{2}{\pi k^2}\sin(kx)\Big|_{x=0}^{\pi} = \frac{2(-1)^{k+1}}{k}.$$

So, $f(x) \sim 2\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\sin(kx)}{k}$.

To find where the series converges, we use the 2π-periodic extension of the odd extension of f:



The series converges to $\begin{cases} x - 2\pi n, \text{ if } & 2\pi n - \pi < x < 2\pi n + \pi,\ n \in \mathbb{Z} \\ 0, \text{ if } & x = (2n+1)\pi,\ n \in \mathbb{Z}. \end{cases}$

2. (4 pt) Find a solution to the following initial/boundary value problem for the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \; 0 < x < \pi, \; t > 0;$$
$$u(0,t) = u(\pi,t) = 0, \; t > 0;$$
$$u(x,0) = 2\sin x, \; \frac{\partial u}{\partial t}(x,0) = \sin(2x), \; 0 < x < \pi.$$

Answer: $u(x,t) = 2\cos t \sin x + \frac{1}{2}\sin(2t)\sin(2x)$.