

Math 54-1
Quiz 12, August 5, 2010

Your name: Key

Please write your name on each sheet. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (5 pt) Find the general solution to the homogeneous equation

$$y'' - 4y' + 13y = 0$$

and determine the form of a particular solution to the inhomogeneous equation

$$y'' - 4y' + 13y = e^{2x}(1 + \cos(3x)) + x^3 \sin(3x)$$

obtained using the method of undetermined coefficients. Do NOT solve for the coefficients.

Auxiliary equation: $r^2 - 4r + 13 = 0$

Roots: $r = 2 \pm \sqrt{4 - 13} = 2 \pm 3i$

Fundamental system: $\{e^{2x} \cos(3x), e^{2x} \sin(3x)\}$

General solution: $c_1 e^{2x} \cos(3x) + c_2 e^{2x} \sin(3x), \quad c_1, c_2 \in \mathbb{R}.$

For $y'' - 4y' + 13y = e^{2x} + e^{2x} \cos(3x) + x^3 \sin(3x)$

e^{2x} solves the hom. eqn, so $x^k \sin(3x), x^k \cos(3x), 0 \leq k \leq 3.$

$x e^{2x} \cos(3x)$
 $x e^{2x} \sin(3x)$

We get the form of a particular solution:

$$Ae^{2x} + Bx e^{2x} \cos(3x) + Cx e^{2x} \sin(3x) +$$

$$+ (Dx^3 + Ex^2 + Fx + G) \sin(3x) +$$

$$+ (Hx^3 + Ix^2 + Jx + K) \cos(3x).$$

2. (5 pt) Find the general solution of the inhomogeneous equation

$$y'' - y' = 1 + e^{-x}.$$

General soln of the homogeneous eqn: $y'' - y' = 0 \rightarrow$
 $\rightarrow y = c_1 + c_2 e^x.$

Trial solution: $y_p = Ax + Be^{-x}$, since 1 is a soln of the homogeneous equation.

$$y_p'' - y_p' = Be^{-x} - (A - Be^{-x}) = 2Be^{-x} - A = 1 + e^{-x} \Rightarrow$$
$$\Rightarrow 2B = 1, A = -1; \text{ so,}$$

$$y_p = -x + \frac{1}{2}e^{-x}.$$

General solution of the inhomogeneous eqn:
 $y = -x + \frac{1}{2}e^{-x} + c_1 + c_2 e^x.$