Solution formulas for certain PDE

For the initial/boundary value problem for the heat equation

$$\frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t),\ 0 < x < \pi,\ t > 0;$$
$$u(0,t) = u(\pi,t) = 0,\ t > 0;$$
$$u(x,0) = f(x),\ 0 < x < \pi,$$

if $f(x)$ has the Fourier sine series

$$f(x) \sim \sum_{k=1}^{\infty} b_k \sin(kx),$$

then the formal solution is

$$u(x,t) = \sum_{k=1}^{\infty} b_k e^{-k^2 t} \sin(kx).$$

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if $f(x)$ has the Fourier cosine series

$$f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(kx),$$

then the formal solution is

$$u(x,t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k e^{-k^2 t}\cos(kx).$$

For the initial/boundary value problem for the wave equation

$$\begin{gathered}
\frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t),\ 0 < x < \pi,\ t > 0;\\
u(0,t) = u(\pi,t) = 0,\ t > 0;\\
u(x,0) = f(x),\ \frac{\partial u}{\partial t}(x,0) = g(x),\ 0 < x < \pi,
\end{gathered}$$

if $f(x)$ and $g(x)$ have the Fourier sine series

$$f(x) \sim \sum_{k=1}^{\infty} b_k \sin(kx),\ g(x) \sim \sum_{k=1}^{\infty} c_k \sin(kx),$$

then the formal solution is

$$u(x,t) = \sum_{k=1}^{\infty} b_k \cos(kt)\sin(kx) + \sum_{k=1}^{\infty} \frac{c_k}{k}\sin(kt)\sin(kx).$$

For the initial/boundary value problem for the wave equation

$$\begin{gathered}
\frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t),\ 0 < x < \pi,\ t > 0;\\
\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0,\ t > 0;\\
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\end{gathered}$$

if $f(x)$ and $g(x)$ have the Fourier cosine series

$$f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(kx),\ g(x) \sim \frac{c_0}{2} + \sum_{k=1}^{\infty} c_k \cos(kx),$$

then the formal solution is

$$u(x,t) = \frac{a_0}{2} + \frac{c_0 t}{2} + \sum_{k=1}^{\infty} b_k \cos(kt)\cos(kx) + \sum_{k=1}^{\infty} \frac{c_k}{k}\sin(kt)\cos(kx).$$