MATH 54-1, H/W 8

5.1 // (14) $\mathrm{Nul}\,(A-\lambda I) = \mathrm{Nul}\,(A+2I) = \mathrm{Nul}\begin{bmatrix} 3 & 0 & -1 \\ 1 & -1 & 0 \\ 4 & -13 & 3 \end{bmatrix} =$

$\overset{\text{(row red)}}{=} \mathrm{Nul}\begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & -1/3 \\ 0 & 0 & 0 \end{bmatrix}$; a basis for this space is $\left\{ \begin{bmatrix} 1/3 \\ 1/3 \\ 1 \end{bmatrix} \right\}$

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(29) Put $\vec{v} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$; then if $A = [\vec{a}_1 \cdots \vec{a}_n]$, we have $A\vec{v} = \vec{a}_1 + \cdots + \vec{a}_n = \begin{bmatrix} s \\ \vdots \\ s \end{bmatrix} = s\cdot\vec{v}$. So, $\vec{v} \neq \vec{0}$ is an eigen~~value~~vector of A with eigenvalue s.

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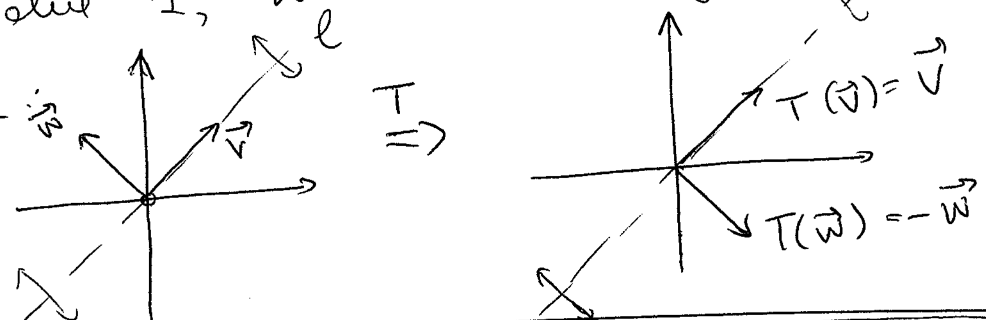
(31) Let $\vec{v}$ lie on the line ℓ, ~~there~~ across which we reflect, and let $\vec{w}$ be orthogonal to this line. Then $T(\vec{v}) = \vec{v}$, $T(\vec{w}) = -\vec{w} \Rightarrow \vec{v}$ is an eigenvector with eigenvalue 1, $\vec{w}$ is an eigenvector with eigenvalue -1;



5.2 // (10) $|A-\lambda I| = \begin{vmatrix} -\lambda & 3 & 1 \\ 3 & -\lambda & 2 \\ 1 & 2 & -\lambda \end{vmatrix} = -\lambda^3 + 14\lambda + 12.$

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(16) $5\,(1)$, $-4\,(1)$, $1\,(2)$; $|A-\lambda I| = (5-\lambda)(-4-\lambda)(1-\lambda)^2$

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(20) $\det(A^T - \lambda I) = \det(A-\lambda I)^T = \det(A-\lambda I)$

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(22) (a) False (forgot about the absolute value: Volume $= |\det|$.
(b) False: $\det A^T = \det A$ (c) True (the § before Example 4)
(d) False: see the warning after Thm 4.

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(24) A, B similar $\Rightarrow \exists$ invertible P: $B = P^{-1}AP \rightarrow$
$\rightarrow \det B = \det P^{-1} \cdot \det A \cdot \det P = \frac{1}{\det P} \cdot \det A \cdot \det P = \det A.$