MATH 54-1, H/W 5

3.1 (10) $\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{vmatrix} = -3 \begin{vmatrix} 1 & -2 & 2 \\ 2 & -6 & 5 \\ 5 & 0 & 4 \end{vmatrix} =$

$-3\left[5\begin{vmatrix} -2 & 2 \\ -6 & 5 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ 2 & -6 \end{vmatrix}\right] = -3(5\cdot 2 - 4\cdot 2) = -6.$

(20) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$; $\begin{vmatrix} a & b \\ kc & kd \end{vmatrix} = a\cdot kd - \cancel{k}\, b\cdot kc = k\begin{vmatrix} a & b \\ c & d \end{vmatrix}.$

Multiplying the second row by k makes the determinant multiply by k.

3.2 (12) Answer: 114 (28) (a) True, see Theorem 3 (doing one row interchange multiplies the determinant by -1)

(b) False; e.g., $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0.0$ columns are linearly indepdt. E.g. $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0$

$\det A = 0 \iff$

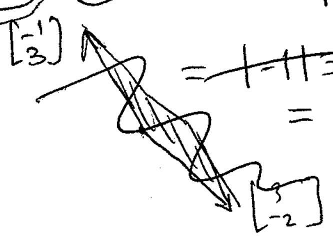
(c) False; $\det(A^T) = \det A$ by Theorem 5.

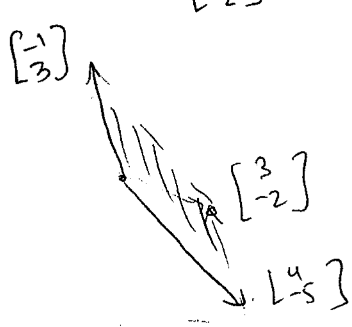
(d) False;

(34) We have $\det(PAP^{-1}) = \det P \cdot \det A \cdot \det(P^{-1}) = \det P \cdot \det A \cdot \frac{1}{\det P} = \det A$.

3.3 (20) Area $= \left|\det\begin{bmatrix} -1 & 4 \\ 3 & -5 \end{bmatrix}\right| = \cancel{|-11| = 1}$

$= |-7| = 7$





(30) Area of R = Area of triangle with vertices $(0,0)$, (x_2-x_1, y_2-y_1), (x_3-x_1, y_3-y_1)

$= \frac{1}{2}\left|\det\begin{bmatrix} x_2-x_1 & x_3-x_1 \\ y_2-y_1 & y_3-y_1 \end{bmatrix}\right|$. Next,

$\frac{1}{2}\left|\det\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}\right| = \frac{1}{2}\left|\det\begin{bmatrix} x_1 & y_1 & 1 \\ x_2-x_1 & y_2-y_1 & 0 \\ x_3-x_1 & y_3-y_1 & 0 \end{bmatrix}\right| =$

$= \frac{1}{2}\left|\det\begin{bmatrix} x_2-x_1 & y_2-y_1 \\ x_3-x_1 & y_3-y_1 \end{bmatrix}\right|$

(first we did row operations and then cofactor expansion)