Math 54-1, H/W 4

2.2

(2) $\begin{bmatrix} 3 & 2 \\ 7 & 4 \end{bmatrix}^{-1} = \frac{1}{12-14}\begin{bmatrix} 4 & -2 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 7/2 & -3/2 \end{bmatrix}$

(8) Given: A invertible, $AD = I$

Need to prove: $D = A^{-1}$.

Since A is invertible, there exists A^{-1}. Multiply both sides of $AD = I$ to the left by A^{-1}:
$A^{-1}AD = A^{-1}$; but $A^{-1}AD = (A^{-1}A)D = I \cdot D = D$.

(10) (a) False: $(AB)^{-1} = B^{-1}A^{-1}$
(b) True, Theorem 6(a)
(c) True, Theorem 4
(d) True, Theorem 7
(e) False, see Theorem 7

(17) $AB = BC$, A, B, C square, there exists B^{-1}.
Multiply by B^{-1} to the ~~left~~ right:
$ABB^{-1} = BCB^{-1}$. But ~~th~~
$A = AI = A(BB)^{-1}$. So, $A = BCB^{-1}$.

2.3

(8) A is already in REF and has 4 pivot positions; therefore, it is invertible.

(15) No, it cannot. If, for simplicity, $A = [\vec{a}_1\ \vec{a}_2\ \vec{a}_3 \dots \vec{a}_n]$ with $\vec{a}_1 = \vec{a}_2$, then the vectors $\{\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n\}$ are linearly dependent, with a linear dependence relation $\vec{a}_1 - \vec{a}_2 = 0$. ~~B~~ It remains to use IMT(e).

(16) No, by IMT(h)

(17) If A is invertible, then A^{-1} is invertible, so the columns of A are lin. ind. by IMT(e)

(22) $H\vec{x} = \vec{c}$ is inconsistent for some $\vec{c} \Rightarrow$ by IMT(g), H is not invertible $\Rightarrow$ by IMT(d), the ~~set~~ ~~$\{A\vec{x}=0\}$~~ equation $H\vec{x} = 0$ has a nonzero solution.

(26) The columns of A are lin. ind. $\Rightarrow$ by IMT(e), A is invertible $\Rightarrow A^2 = A \cdot A$ is invertible $\Rightarrow$ by IMT(h), the columns of A span $\mathbb{R}^4$.