Math 54-1, H/W2

1.4. 6) $-2\begin{bmatrix}7\\2\\9\\-3\end{bmatrix}-5\begin{bmatrix}-3\\1\\-6\\2\end{bmatrix}=\begin{bmatrix}1\\-9\\12\\-4\end{bmatrix}$

22) Put $A=[\vec{v}_1\ \vec{v}_2\ \vec{v}_3]=\begin{bmatrix}0&0&4\\0&-3&-1\\-2&8&-5\end{bmatrix}\xrightarrow[\text{red}]{\text{row}}\begin{bmatrix}\boxed{-2}&8&-5\\0&\boxed{-3}&-1\\0&0&\boxed{4}\end{bmatrix}$

has a pivot in each row $\to$ its columns $\{\vec{v}_1,\vec{v}_2,\vec{v}_3\}$ span $\mathbb{R}^3$.

30) Idea: start with a matrix in REF with this property and transform it by row operations (which do not change pivot positions and thus the fact that the columns do not span $\mathbb{R}^3$).

For example, $A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}$ does not have a pivot in the last row;

$A\xrightarrow{R_3\leftarrow R_3+R_1}\begin{bmatrix}1&0&0\\0&1&0\\1&0&0\end{bmatrix}$ ~~is an example.~~ has the required property.

1.5. 24) a) False; e.g. $A=\begin{bmatrix}1&0\\0&0\end{bmatrix}$, $\vec{x}=\begin{bmatrix}0\\1\end{bmatrix}$

b) True, see Example 2 c) True; if $\vec{x}=\vec{0}$ is a solution, then $\vec{b}=A\vec{x}=A\vec{0}=\vec{0}$. d) True; see the paragraph after example 3

e) False, since the solution set of $A\vec{x}=\vec{b}$ can be empty.

1.7 3) Put $A=\begin{bmatrix}1&-3\\-3&9\end{bmatrix}$, then we row reduce A

to $\begin{bmatrix}\boxed{1}&-3\\0&0\end{bmatrix}$, no pivot in column 2 – Lin. Dep.

Also, $\begin{bmatrix}-3\\9\end{bmatrix}=3\begin{bmatrix}1\\-3\end{bmatrix}$.

⑥ $\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix} \xrightarrow[\text{red}]{\text{row}} \begin{bmatrix} \boxed{1} & 0 & 3 \\ 0 & \boxed{-1} & 4 \\ 0 & 0 & \boxed{7} \\ 0 & 0 & 0 \end{bmatrix}$ Pivot in every column $\Rightarrow$ $\rightarrow$ LIN. IND.

⑯ LIN. ~~IND~~. DEP., as the second vector $= \frac{3}{2} \cdot$ the first vector

⑰ LIN. DEP., as contain the zero vector. (Th. 9)