MATH 54-1, H/W 12

4.2 (2) $y'' - y' - 2y = 0$. Auxiliary equation: $r^2 - r - 2 = 0$, roots $r = -1, 2$. Fundamental system $\{e^{-x}, e^{2x}\}$. General solution: $c_1 e^{-x} + c_2 e^{2x}$; $c_1, c_2 \in \mathbb{R}$.

(26) $y'' + y = 0 \to$ general solution $y = c_1 \cos x + c_2 \sin x$.

(a) $\begin{cases} 2 = y(0) = c_1 \\ 0 = y(\pi/2) = c_2 \end{cases} \to c_1 = 2, c_2 = 0$, unique solution

(b) $\begin{cases} 2 = y(0) = c_1 \\ 0 = y(\pi) = -c_1 \end{cases} \to$ no solution

(c) $\begin{cases} 2 = y(0) = c_1 \\ -2 = y(\pi) = -c_1 \end{cases} \to c_1 = 2$, c_2 free

Infinitely many solutions

4.3 (2) $y'' + y = 0 \to r^2 + 1 = 0 \to r = \pm i \to \{\cos x, \sin x\}$.

General solution: $c_1 \cos x + c_2 \sin x$; $c_1, c_2 \in \mathbb{R}$.

(24) $y'' + 9y = 0 \to y = c_1 \cos(3x) + c_2 \sin(3x)$.

$y' = -3c_1 \sin(3x) + 3c_2 \cos(3x)$

$\begin{cases} 1 = y(0) = c_1 \\ \cancel{10} = y'(0) = 3c_2 \end{cases} \to c_1 = 1, c_2 = 1/3, y = \cos(3x) + \frac{1}{3}\sin(3x)$.

4.4 (2) Yes (4) Yes; $(\sin x)/e^{4x} = e^{-4x}\sin x$ (16) $\theta'' - \theta = t\sin t$.

Homogeneous equation: $\theta'' - \theta = 0 \to$ general solution $c_1 e^t + c_2 e^{-t}$.

Trial solution: $\theta_p = (At + B)\sin t + (Ct + D)\cos t$.

$\theta_p' = (At + B + C)\cos t - (Ct + D - A)\sin t$

$\theta_p'' = -(At + B + 2C)\sin t - (Ct + D - 2A)\cos t$ ~~$=$~~

~~$\theta_p'' = \theta_p = -(B+2C)\sin t - (D - 2A)\cos t =$~~

$\theta_p'' - \theta_p = -2(At + B + C)\sin t - 2(Ct + D - A)\cos t =$

$t\sin t \to \begin{cases} -2A = 1 & [t\sin t] \\ B + C = 0 & [\sin t] \\ C = 0 & [t\cos t] \\ D - A = 0 & [\cos t] \end{cases}$

$A = -\frac{1}{2}, B = 0, C = 0, D = -\frac{1}{2}$

$\boxed{\theta_p = -\frac{1}{2} t\sin t - \frac{1}{2}\cos t}$

(30) $y'' - 2y' + y = 7e^t \cos t$. General solution of the homogeneous equ: $y'' - 2y' + y = 0 \rightarrow y = \cancel{c_1 e^t \cos t + c_2 e^t \sin t}$ $c_1 e^t + c_2 t e^t$.

$e^t \cos t$ does not solves the hom. equ $\Rightarrow$

$\Rightarrow$ trial solution: $e^t(A\cos t + B \sin t)$.

4.5 (10) $y'' - y' + y = (e^t + t)^2 = e^{2t} + 2t \cdot e^t + t^2$

~~Trial solutions:~~ The method of undetermined coefficients can be applied

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(20) $y'' + 4y = \sin\Theta - \cos\Theta \longrightarrow$ trial solution:

$y_p = A\cos\Theta + B\sin\Theta \rightarrow y_p'' + 4y_p = 3A\cos\Theta + 3B\sin\Theta =$

$= \sin\Theta - \cos\Theta \Rightarrow 3A = -1, \quad 3B = 1 \rightarrow A = -\frac{1}{3}, \; B = \frac{1}{3};$

$y_p = -\frac{1}{3}\cos\Theta + \frac{1}{3}\sin\Theta$. General solution to the homogeneous equ: $c_1\cos(2\Theta) + c_2 \sin(2\Theta)$;

general solution to the inhomogeneous equ:

$y = -\frac{1}{3}\cos\Theta + \frac{1}{3}\sin\Theta + c_1\cos(2\Theta) + c_2\sin(2\Theta); \; c_1, c_2 \in \mathbb{R}$.

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(34) $y'' + 5y' + 6y = \sin t - \cos 2t$. General solu to the homogeneous equ: $c_1 e^{-2t} + c_2 e^{-3t}$. Trial solution:

$y_p = A\sin t + B\cos t + C\cos 2t + D\sin 2t$.