MATH 54-1, H/W 11

6.4 (4) $\vec{v}_1 = \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -3 \\ 14 \\ -7 \end{bmatrix} - \dfrac{-9-56-35}{9+16+25} \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$.

(8) $\vec{u}_1 = \dfrac{\vec{v}_1}{|\vec{v}_1|} = \dfrac{1}{5\sqrt{2}} \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}$, $\vec{u}_2 = \dfrac{\vec{v}_2}{|\vec{v}_2|} = \dfrac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

(22) Recall that

$T(\vec{x}) = \dfrac{\vec{x}\cdot\vec{u}_1}{\|\vec{u}_1\|^2}\vec{u}_1 + \cdots + \dfrac{\vec{x}\cdot\vec{u}_p}{\|\vec{u}_p\|^2}\vec{u}_p$. For any $\vec{x}, \vec{y} \in \mathbb{R}^n$, $c, d \in \mathbb{R}$,

$$T(c\vec{x}+d\vec{y}) = \frac{(c\vec{x}+d\vec{y})\cdot\vec{u}_1}{\|\vec{u}_1\|^2}\vec{u}_1 + \cdots + \frac{(c\vec{x}+d\vec{y})\cdot\vec{u}_p}{\|\vec{u}_p\|^2}\vec{u}_p =$$

$$= c\frac{\vec{x}\cdot\vec{u}_1}{\|\vec{u}_1\|^2}\vec{u}_1 + d\frac{\vec{y}\cdot\vec{u}_1}{\|\vec{u}_1\|^2}\vec{u}_1 + \cdots + c\frac{\vec{x}\cdot\vec{u}_p}{\|\vec{u}_p\|^2}\vec{u}_p + d\frac{\vec{y}\cdot\vec{u}_p}{\|\vec{u}_p\|^2}\vec{u}_p =$$

$$= c\left(\frac{\vec{x}\cdot\vec{u}_1}{\|\vec{u}_1\|^2}\vec{u}_1 + \cdots + \frac{\vec{x}\cdot\vec{u}_p}{\|\vec{u}_p\|^2}\vec{u}_p\right) + d\left(\frac{\vec{y}\cdot\vec{u}_1}{\|\vec{u}_1\|^2}\vec{u}_1 + \cdots + \frac{\vec{y}\cdot\vec{u}_p}{\|\vec{u}_p\|^2}\vec{u}_p\right) =$$

$= c\,T(\vec{x}) + d\,T(\vec{y})$. Therefore, $T$ is linear.

6.5 (2) Normal system: $\begin{bmatrix} 12 & 8 \\ 8 & 10 \end{bmatrix} \hat{x} = \begin{bmatrix} -24 \\ -2 \end{bmatrix} \Rightarrow \hat{x} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$.

(6) Normal system has augmented matrix $\begin{bmatrix} 6 & 3 & 3 & 27 \\ 3 & 3 & 0 & 12 \\ 3 & 0 & 3 & 15 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$; general least-squares solution: $\hat{x} = \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$.

6.7 (3) $\langle p, q\rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1) = 3\cdot 1 + 4\cdot 5 + 5\cdot 1 = 28$.

(5) $\langle p, p\rangle = p(-1)^2 + p(0)^2 + p(1)^2 = 50 \rightarrow \|p\| = 5\sqrt{2}$

$\langle q, q\rangle = q(-1)^2 + q(0)^2 + q(1)^2 = 27 \rightarrow \|q\| = 3\sqrt{3}$

(21) $\langle f, g\rangle = \int_0^1 (1-3t^2)(t-t^3)\,dt = \int_0^1 3t^5 - 4t^3 + t\,dt = 0$

(23) $\|f\|^2 = \langle f, f\rangle = \int_0^1 (1-3t^2)^2\,dt = 4/5 \rightarrow \|f\| = 2/\sqrt{5}$.