Math 54, Section 214 Your name: Key

Quiz 1, September 29, 2009

Please write your name **on each sheet**. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (6 pt) Find the general solution of the system whose augmented matrix is

$$\begin{bmatrix} 1 & -1 & 0 & -2 \\ 2 & 0 & 1 & -1 \\ 1 & -3 & -1 & -5 \end{bmatrix}.$$

Show the intermediate steps of row reduction. Indicate which variables are basic and which are free.

Row reduction:

$$\begin{bmatrix} 1 & -1 & 0 & -2 \\ 2 & 0 & 1 & -1 \\ 1 & -3 & -1 & -5 \end{bmatrix} \xrightarrow[R_3 \leftarrow R_3 - R_1]{R_2 \leftarrow R_2 - 2R_1} \begin{bmatrix} 1 & -1 & 0 & -2 \\ 0 & 2 & 1 & 3 \\ 0 & -2 & -1 & -3 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 + R_2}$$

$$\longrightarrow \begin{bmatrix} \boxed{1} & -1 & 0 & -2 \\ 0 & \boxed{2} & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

System is consistent

Basic variables: x_1, x_2

Free variable: x_3

$$\begin{cases} x_1 - x_2 = -2 \\ 2x_2 + x_3 = 3 \end{cases} \Rightarrow \begin{cases} x_1 - x_2 = -2 \\ x_2 = \frac{3}{2} - \frac{1}{2}x_3 \end{cases} \rightarrow \boxed{\begin{array}{l} x_1 = -\frac{1}{2} - \frac{1}{2}x_3 \\ x_2 = \frac{3}{2} - \frac{1}{2}x_3 \\ x_3 \text{ free} \end{array}}$$

2. (6 pt) Does v lie in Span{u_1, u_2}, where

$$v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad u_1 = \begin{bmatrix} -2 \\ -4 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 3 \\ -6 \end{bmatrix}?$$

Explain.

$\vec{v}$ lies in Span$\{\vec{u}_1, \vec{u}_2\}$ $\Leftrightarrow$

$\Leftrightarrow$ the system with the augmented matrix $[\vec{u}_1 \ \vec{u}_2 \ \vec{v}]$ is consistent.

Row reduction: $[\vec{u}_1 \ \vec{u}_2 \ \vec{v}] = \begin{bmatrix} -2 & 3 & 1 \\ -4 & -6 & 2 \end{bmatrix} \Rightarrow$

$\Rightarrow \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{bmatrix} \boxed{-2} & 3 & 1 \\ 0 & \boxed{-12} & 0 \end{bmatrix} \to$ consistent

Answer: $\vec{v}$ lies in Span$\{\vec{u}_1, \vec{u}_2\}$.

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3. (7 pt) Do the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

span $\mathbb{R}^2$? Explain.

$\vec{v}_1, \dots, \vec{v}_3$ span $\mathbb{R}^2 \overset{\text{Th. 4}}{\iff} A = \begin{bmatrix} 0 & 0 & 4 \\ 2 & 1 & 6 \end{bmatrix}$

has a pivot position in every row.

~~Change~~ r Swap rows 1 and 2:

$A \to \begin{bmatrix} \boxed{2} & 1 & 6 \\ 0 & 0 & \boxed{4} \end{bmatrix}$ has a pivot in every row

Answer: the vectors $\vec{v}_1$, ~~and~~ $\vec{v}_2, \vec{v}_3$ span $\mathbb{R}^2$.