Math 54, Section 214
Quiz 10, April 23, 2010

Your name: Key

Please write your name on each sheet. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (10 pt) Find a fundamental system $\{y_1(x), y_2(x), y_3(x)\}$ for the equation

$$y''' - y' = 0.$$

Compute the Wronskian $W(y_1, y_2, y_3)$ for your favourite value of x and show that $\{y_1, y_2, y_3\}$ is linearly independent.

Auxiliery equation: $r^3 - r = 0 \rightarrow r(r^2-1) = 0$

Roots $\lambda = 0, \pm 1;$

Fundamental system: $y_1 = 1, \; y_2 = e^x, \; y_3 = e^{-x}$

Wronskian: $W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} = \begin{vmatrix} 1 & e^x & e^{-x} \\ 0 & e^x & -e^{-x} \\ 0 & e^x & e^{-x} \end{vmatrix}.$

Put $x = 0 \rightarrow \quad W = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2 \neq 0;$

Therefore, $\{y_1, y_2, y_3\}$ are linearly independant.

2. (10 pt) Write the system

$$x_1'' = tx_1 + x_2' + \cos t,$$
$$x_2'' = tx_2 - 2x_1' - e^t$$

in the normal form; i.e., in the form

$$\vec{x}'(t) = A(t)\vec{x}(t) + \vec{f}(t),$$

where $\vec{x}$ and $\vec{f}$ are certain vectors and A is a certain matrix.

Put $\vec{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_1' \\ X_2' \end{bmatrix}$; then $\vec{X}' = \begin{bmatrix} X_1' \\ X_2' \\ X_1'' \\ X_2'' \end{bmatrix} =$

$$= \begin{bmatrix} X_1' \\ X_2' \\ tX_1 + X_2' + \cos t \\ tX_2 - 2X_1' - e^t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ t & 0 & 0 & 1 \\ 0 & t & -2 & 0 \end{bmatrix} \vec{X} + \begin{bmatrix} 0 \\ 0 \\ \cos t \\ -e^t \end{bmatrix}.$$