

Math 54, Section 214
Quiz 9, April 16, 2010

Your name: Key

Please write your name on each sheet. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (a) (6 pt) Find the general solution of the homogeneous equation

$$y'' - 4y' + 13y = 0.$$

- (b) (4 pt) Determine the form of a particular solution to each of the equations below (obtained using the method of undetermined coefficients). Do NOT solve.

$$y'' - 4y' + 13y = e^{2x}(1 + \cos(3x)),$$

$$y'' - 4y' + 13y = x^3 \sin(3x).$$

(a) Auxiliary equation: $r^2 - 4r + 13 = 0 \Rightarrow r = 2 \pm \sqrt{4-13} = 2 \pm 3i.$

$$y = c_1 e^{2x} \cos(3x) + c_2 e^{2x} \sin(3x).$$

(b) $f = e^{2x}(1 + \cos 3x) \rightarrow$

$$\rightarrow y_p = Ae^{2x} + Bxe^{2x} \cos(3x) + Cxe^{2x} \sin(3x)$$

$$f = x^3 \sin(3x) \rightarrow$$

$$\rightarrow y_p = (Ax^3 + Bx^2 + Cx + D) \cos(3x) + (Ex^3 + Fx^2 + Gx + H) \sin(3x),$$

2. (a) (7 pt) Find the general solution to the inhomogeneous equation

$$y'' - y' = 1 + e^{-x}.$$

(b) (3 pt) Find the solution to the equation above satisfying the initial conditions

$$y(0) = 0, y'(0) = 1.$$

(a) General solution to the homogeneous eqn. :
 $y'' - y' = 0 \rightarrow r^2 - r = 0 \rightarrow r = 0, 1 \rightarrow$
 $\rightarrow y = c_1 + c_2 e^x.$

Trial solution: $y_p = Ax + Be^{-x}.$

$$y_p' = A - Be^{-x}, y_p'' = Be^{-x},$$

$$y_p'' - y_p' = -A + 2Be^{-x} = 1 + e^{-x} \rightarrow A = -1$$
$$B = \frac{1}{2}$$

$$y_p = -x + \frac{1}{2} e^{-x}.$$

General solution to the inhomogeneous eqn.:

$$y = -x + \frac{1}{2} e^{-x} + c_1 + c_2 e^x$$

(b)

$$0 = y(0) = \frac{1}{2} + c_1 + c_2 \quad \left| \quad c_1 + c_2 = -\frac{1}{2} \rightarrow c_1 = -3$$
$$1 = y'(0) = -1 - \frac{1}{2} + c_2 \quad \left| \quad c_2 = \frac{5}{2}$$

So,

$$y = -x + \frac{1}{2} e^{-x} - 3 + \frac{5}{2} e^x.$$