Math 1B practice midterm *

Sep 27, 2009

1. (10%) Compute the integral $\int_0^1 \arccos x \, dx$.

2. (15%) If a is a positive constant, compute the integral $\int x^3\sqrt{a^2+x^2}\, dx$.

3. (15%) Compute the integral $\int \frac{4\, dx}{4+e^{2x}}$.

4–5. (10%) Write each of the following functions as the sum of a polynomial and partial fractions, but do not try to determine the numerical values of the coefficients in the latter:

$$\frac{2x^5-x-1}{x^4+6x^2+9},$$
$$\frac{65x^3+28x+47}{(x^2-5)(x^2-4x+12)}.$$

6. (15%) Let f be a function defined on an interval $[a, b]$ and let the fourth derivative $f^{(4)}$ of f satisfy $|f^{(4)}(x)| \leqslant K$ for $a \leqslant x \leqslant b$. If E_s is the error involved in using Simpson's rule with n subdivisions (n being even), then it is known that

$$|E_s| \leqslant \frac{K(b-a)^5}{180n^4}.$$

(a) Suppose that f is a function defined on $[0, 4]$ so that $|f^{(4)}(x)| \leqslant 1$ for all x in $[0, 4]$, and so that $f(0) = 2, f(1) = 1, f(2) = -1.3, f(3) = -3.6$, and $f(4) = -3.3$. Use the given data and Simpson's rule to approximate $\int_0^4 f(x)\, dx$ to one decimal place.

(b) Estimate the error of this approximation to *two* decimal places.

(c) What is the range of possible values of $\int_0^4 f(x)$ according to (a) and (b) above?

7. (10%) Does the series $\sum_{n=1}^{\infty} \frac{7n^2}{7+n^2}$ converge?

8. (15%) Let $a_n = \frac{n!}{2^n}$. Find the limit $\lim_{n\to\infty} \frac{a_{n+2}}{a_n}(1-\cos(1/n))$.

9. (10%) Find the centroid of the region bounded by the curves

$$y = x^3,\ x+y = 2,\ y = 0.$$

*Problems 1–6 are taken from Math 1B first midterm in Fall 2002 semester by Prof. Hung-Hsi Wu. Problem 9 is taken from a recent quiz by Claudiu Raicu.

Hints and answers

1. Integrate by parts with $u = \arccos x$, $dv = dx$. Then make the change of variables $t = x^2$.

Answer: $x \arccos x - \sqrt{1 - x^2} + C$.

2. Make the change of variables $u = a^2 + x^2$.

Answer: $\left(\frac{x^2}{5} - \frac{2a^2}{15}\right)(a^2 + x^2)^{3/2} + C$.

3. Make the change of variables $u = e^{2x}$, then integrate by partial fractions.

Answer: $x - \frac{1}{2} \ln(4 + e^{2x}) + C$.

4. Answer: $2x + \frac{Ax+B}{x^2+3} + \frac{Cx+D}{(x^2+3)^2}$.

5. Answer: $\frac{A}{x-\sqrt{5}} + \frac{B}{x+\sqrt{5}} + \frac{Cx+D}{x^2-4x+12}$.

6. (a) Answer: -14.3.

(b) Answer: $|E_s| \leqslant 0.03$. Note that we had to round up $\frac{1}{45}$ here!

(c) Answer: $-14.33 \leqslant \int \leqslant -14.27$.

7. Dividing the numerator and the denominator by n^2, we get $\lim_{n\to\infty} \frac{7n^2}{7+n^2} = 7$. Since this is nonzero, the series diverges.

8. We have $\frac{a_{n+2}}{a_n}(1 - \cos(1/n)) = (n+2)(n+1)(1 - \cos(1/n))/4 = f(1/n)$, where $f(x) = \frac{(2x+1)(x+1)(1-\cos x)}{4x^2}$. We then use that $\lim_{x\to 0}(2x+1)(x+1) = 1$ and compute $\lim_{x\to 0} \frac{1-\cos x}{x^2}$ by applying L'Hôpital's Rule twice.

Answer: $\frac{1}{8}$.

9. See the solution to problem 3 in quiz 4 on the webpage

`http://math.berkeley.edu/~claudiu/math1b.html`

Answer: $\left(\frac{52}{45}, \frac{20}{63}\right)$.