Math 1B, Section 106 Your name: Key

Quiz 8, October 28, 2009

Please write your name **on each sheet**. Show your work clearly and in order, including the intermediate steps in the solutions and the final answer.

1. (4 pt) Approximate the function $f(x) = 1/x$ near the point $a = 1$ using the Taylor polynomial T_1. Estimate the error of approximation on the interval $[1/3, 4/3]$ using Taylor's inequality. The answer should be a number independent of x.

$f(x) = 1/x$, $f'(x) = -\frac{1}{x^2}$, $f'''(x) = \frac{2}{x^3}$.

$f(a) = 1$, $f'(a) = -1$;

$T_1(x) = 1 - 1(x-1) = \boxed{2-x}$

Taylor's inequality: $|f(x) - T_1(x)| \leq \frac{M|x-1|^2}{2}$,

where $M \geq |f''(t)|$ for $t \in [\frac{1}{3}, \frac{4}{3}]$.

$|f''(t)| = \frac{2}{t^3}$ is decreasing $\rightarrow$ can take

$M = |f''(\frac{1}{3})| = \frac{2}{(1/3)^3} = 54$.

For $x \in [\frac{1}{3}, \frac{4}{3}]$, $|x-1| \leq \frac{2}{3}$; So, $|x-1|^2 \leq \frac{4}{9}$ and

$\boxed{|f(x) - T_1(x)| \leq 12}$

2. (6 pt) (a) Find the first nonzero term (the nonzero term with the least power of x) of the Maclaurin series for the function

$$f(x) = \frac{e^{-x}}{1-x} - 1.$$

You may use the formula $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

(b) Find an integer $k \geqslant 0$ such that the limit $\lim_{x\to 0} \frac{f(x)}{x^k} = c$ is a finite nonzero number. Evaluate c.

(a) $\frac{1}{1-x} = 1 + x + x^2 + \cdots$

$e^{-x} = 1 - x + \frac{x^2}{2} + \cdots$

$f(x) = (1 - x + \frac{x^2}{2} + \cdots)(1 + x + x^2 + \cdots) - 1 =$

$= 1 - x + x + \frac{x^2}{2} - x^2 + x^2 + \cdots - 1 =$

$= \frac{x^2}{2} + \cdots$, where $(\cdots)$ has powers $\geqslant 3$ of x.

Leading term: $\boxed{\frac{x^2}{2}}$

(b) Put $k = 2$, then $\lim_{x\to 0} \frac{f(x)}{x^{\cancel{k}2}} =$

$= \lim_{x\to 0} (\frac{1}{2} + (\cdots)) = \frac{1}{2}$ because $(\cdots)$ has powers $\geqslant 1$ of x.

$\boxed{k = 2 \quad c = \frac{1}{2}}$