Math 1B, Section 106 Your name: Key

Quiz 7, October 14, 2009

Please write your name on each sheet. Show your work clearly and in order, including the intermediate steps in the solutions and the final answer.

1. (5 pt) Consider the series

$$\sum_{n=0}^{\infty} (-1)^n \sqrt{n+1} \cdot x^n.$$

(a) Find the radius of convergence and the interval of convergence.

(b) Integrate the series to obtain a new series.

(c) Find the interval of convergence of the integrated series. (You may use that this series has the same radius of convergence as the original series.)

(a) Ratio Test: $\lim_{n\to\infty} \left| \frac{(-1)^{n+1}\sqrt{n+2}\,x^{n+1}}{(-1)^n \sqrt{n+1}\,x^n} \right| = \lim_{n\to\infty} |x| \sqrt{1+\frac{1}{n+1}} = |x|.$

$|x|<1 \to$ Converges

$|x|>1 \to$ Diverges

$\} \to \boxed{R=1}$

Endpoints:

$x=1$: $\sum_{n=0}^{\infty} (-1)^n \sqrt{n+1}$ | $x=-1$: $\sum_{n=0}^{\infty} \sqrt{n+1}$

Both diverge by Test for Divergence $\to$ Interval of Convergence: $(-1,1)$

(b) ~~Differ~~ Integrate:

$$\int \sum_{n=0}^{\infty} (-1)^n \sqrt{n+1}\cdot x^n \, dx =$$

$$= C + \sum_{n=0}^{\infty} (-1)^n \sqrt{n+1}\cdot \frac{x^{n+1}}{n+1} =$$

$$= C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{\sqrt{n+1}}$$

(c) $\boxed{R=1}$, same as for the original series

Endpoints:

$x=1$: $C + \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

Converges by A.H. Series Test: $\frac{1}{\sqrt{n+2}} \le \frac{1}{\sqrt{n+1}}$, $\frac{1}{\sqrt{n+1}} \xrightarrow[n\to\infty]{} 0$.

$x=-1$: $C + \sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1}}$

Diverges by p-series test

Interval of Convergence: $(-1,1]$ $\Leftarrow$

2. (5 pt) Find a power series representation (in terms of powers of x) of the function

$$f(x) = \ln\left(\frac{1+x}{1-x}\right).$$

Determine the radius of convergence of the series.

$$f'(x) = (\ln(1+x) - \ln(1-x))' = \frac{1}{1+x} + \frac{1}{1-x} =$$

$$= \frac{(1-x)+(1+x)}{1-x^2} = \frac{2}{1-x^2} = 2\cdot\frac{1}{1-x^2} =$$

$$= 2\sum_{n=0}^{\infty}(x^2)^n = 2\sum_{n=0}^{\infty}x^{2n},$$

converges for $|x^2|<1 \Leftrightarrow |x|<1$

$\Downarrow$

$\boxed{R=1}$

Next, $f(x) = \int f'(x)\,dx =$

$$= C + 2\sum_{n=0}^{\infty}\int x^{2n}\,dx =$$

$$= C + 2\sum_{n=0}^{\infty}\frac{x^{2n+1}}{2n+1}.$$

Substitute $x=0$:

$$0 = f(0) = C + 2\sum_{n=0}^{\infty}\frac{0^{2n+1}}{2n+1} =$$

$$= C \Rightarrow \underline{C=0}$$

$$\boxed{f(x) = 2\sum_{n=0}^{\infty}\frac{x^{2n+1}}{2n+1}}$$

Radius of convergence is the same as for the series for $f'(x)$.