Math 1B worksheet

Nov 23, 2009

1–3. Find two linearly independent solutions to the given second order differential equation. Verify that they solve the equation. Use these two solutions to write the general form of any solution to the equation:

$$y'' + y = 0; \tag{1}$$
$$y'' - y = 0; \tag{2}$$
$$y'' = 0. \tag{3}$$

4–5. Solve the initial value problems:

$$y'' + 3y' + 2y = 0,\ y(0) = 0,\ y'(0) = 1; \tag{4}$$
$$y'' - 2y' + y = 0,\ y(0) = 1,\ y'(0) = 0. \tag{5}$$

6–7. Find all solutions to the boundary-value problems:

$$y'' - y = 0,\ y(-1) = -1,\ y(1) = 1; \tag{6}$$
$$y'' - 2y' + 2y = 0,\ y(0) = y(\pi) = 0. \tag{7}$$

8–10. Find the general solution to the inhomogeneous equations:

$$y'' + 9y = e^{3x}, \tag{8}$$
$$y'' - 2y' + y = \frac{e^x}{1+x^2}, \tag{9}$$
$$y'' + y = \cos x + x\cos 2x. \tag{10}$$

11–12. Solve the initial or boundary value problems:

$$y'' - y = e^x,\ y(0) = y'(0) = 0; \tag{11}$$
$$y'' + y = e^x,\ y(0) = y(\pi/2) = 0. \tag{12}$$

Hints and answers

1. $y = \cos x$ and $y = \sin x$; $y = c_1 \cos x + c_2 \sin x$ is the general solution.
2. $y = e^x$ and $y = e^x$; $y = c_1 e^x + c_2 e^{-x}$ is the general solution.
3. $y = 1$ and $y = x$; $y = c_1 + c_2 x$ is the general solution.
4. $y = e^{-x} - e^{-2x}$.
5. $y = e^x - xe^x$.
6. $y = \frac{e^x - e^{-x}}{e - e^{-1}}$.
7. $y = Ce^x \sin x$. (Note that the solution is not unique!)
8. The trial solution is $y = Ae^{3x}$; we get $y = \frac{e^{3x}}{18} + c_1 \cos(3x) + c_2 \sin(3x)$.
9. $y = -\frac{1}{2} \ln(1 + x^2)e^x + xe^x \arctan x + c_1 e^x + c_2 xe^x$.
10. The trial solution is $y = x(A \cos x + B \sin x) + (Cx + D) \cos(2x) + (Ex + F) \sin(2x)$; we get $y = \frac{1}{2}x \sin x - \frac{1}{3}x \cos(2x) + \frac{4}{9} \sin(2x) + c_1 \cos x + c_2 \sin x$.
11. $y = \frac{1}{2}xe^x + \frac{1}{4}e^{-x} - \frac{1}{4}e^x$.
12. $y = \frac{e^x}{2} - \frac{1}{2} \cos x - \frac{e^{\pi/2}}{2} \sin x$.