WHAT'S NEEDED FROM COMPLEX ANALYSIS IN 18.155

In 18.155 we will use some complex analysis, but not too much (e.g. no Riemann Mapping Theorem). Here is what we will need. One possible source for the below is Ahlfors's Complex Analysis textbook.

- Complex exponential: e^z for $z \in \mathbb{C}$. For $t > 0$ and $z \in \mathbb{C}$, define $t^z := \exp(z \log t)$.
- Complex log: defined e.g. on $\mathbb{C} \setminus (-\infty, 0]$.
- The definition of a holomorphic function on an open set $U \subset \mathbb{C}$: 'complex differentiable'.
- Basic properties of holomorphic functions e.g. Cauchy's formula
$$\oint_\gamma f(z)\, dz = 0$$
if γ is a closed contour bounding an open set where f is holomorphic, and the other Cauchy's formula
$$f(w) = \frac{1}{2\pi i} \oint_\gamma \frac{f(z)}{z - w}\, dz$$
where f is holomorphic in a neighborhood of z and γ is a contour in that neighborhood going around z in the counterclockwise direction. Corollary of the latter formula: every holomorphic function is actually C^∞.
- The fact that each holomorphic function f in a disk $B(w, r)$ is the sum of the Taylor series of that function centered at w. This implies that if $f^{(k)}(w) = 0$ for all k then $f = 0$ on $B(w, r)$. And this gives unique continuation: if f is holomorphic in a connected open set $U \subset \mathbb{C}$ and $f = 0$ on some ball inside U (in fact, enough to assume that all derivatives of f vanish at a single point) then $f = 0$ on the whole U.
- Isolated singularities of holomorphic functions: we only need to deal with poles. Laurent expansion at a pole and the definition of residue. The residue theorem for computing the integral
$$\oint_\gamma f(z)\, dz$$
where f is holomorphic on $U \setminus \{z_1, \dots, z_m\}$, f has poles at $z_1, \dots, z_m$, and γ is a closed contour in U enclosing some of the poles but not passing through any of them.