18.700 Problem Set 2

Due in class Monday September 23; late work will not be accepted. Your work on graded problem sets should be written entirely on your own, although you may consult others before writing.

1. (3 points) Let V be the vector space of polynomials of degree at most five with real coefficients. Define a linear map

$$T: V \to \mathbb{R}^3, \quad T(p) = (p(1), p(2), p(3)).$$

That is, the coordinates of the vector $T(p)$ are the values of p at 1, 2, and 3.

a) Find a basis of the null space of T.

b) Find a basis of the range of T.

2. (3 points) Let V be the vector space of polynomials of degree at most 999 with real coefficients. Define a linear map

$$T: V \to \mathbb{R}^{100}, \quad T(p) = (p(1), p(2), \ldots, p(100)).$$

a) Find the dimension of the null space of T.

b) Find the dimension of the range of T.

3. (6 points) Let V be the vector space of polynomials of degree at most 99 with real coefficients. Define a linear map

$$T: V \to \mathbb{R}^{1000}, \quad T(p) = (p(1), p(2), \ldots, p(1000)).$$

a) Find the dimension of the null space of T.

b) Find the dimension of the range of T.

c) (Harder.) Is the vector $(-1, 1, -1, 1, -1, 1, \ldots, -1, 1)$ in the range of T? That is, is there a polynomial of degree at most 99 whose values at $1, 2, \ldots, 1000$ alternate between -1 and 1?

4. (3 points) Axler, page 49, exercise 14.

5. (5 points) Axler, page 49, exercise 17.