Gambling Tactics: Cashing In On Applied Math

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SPAMS 5/11

Overview

(Disclaimer: We are only considering games for which the outcome is unaffected by human choices. Sorry, no blackjack/poker!!)

1. **Customizing Outcomes**
2. **Craps**
3. **Roulette**

Betting Systems

- Definition: Any strategy one employs to decide how much money should be wagered on a given play in an effort to maximize winnings.

- Typical betting system variables include:
  - Current bank-roll
  - Past losses/wins
  - Number of previous bets
  - Superstition

Fallacy of Betting Systems

- **Fact 1**: The outcome of each play is independent of all others.

  For one play, define:

$$\text{Expected Gain} \equiv E\left(\frac{\text{Winnings}}{\text{Amount Bet}}\right)$$

$$= [\text{Payout Ratio}]P(\text{winning}) - P(\text{losing})$$

- **Fact 2**: The expected gain is independent of the amount bet.

THUS:

If the payout ratio remain constant, the expected total winnings associated to a sequence of wagers

$$B_1, B_2, \ldots, B_N$$

is

$$(B_1 + B_2 + \ldots + B_N)(\text{Expected Gain}).$$

Betting systems cannot increase the expected amount of winnings.

What Betting Systems Can Do

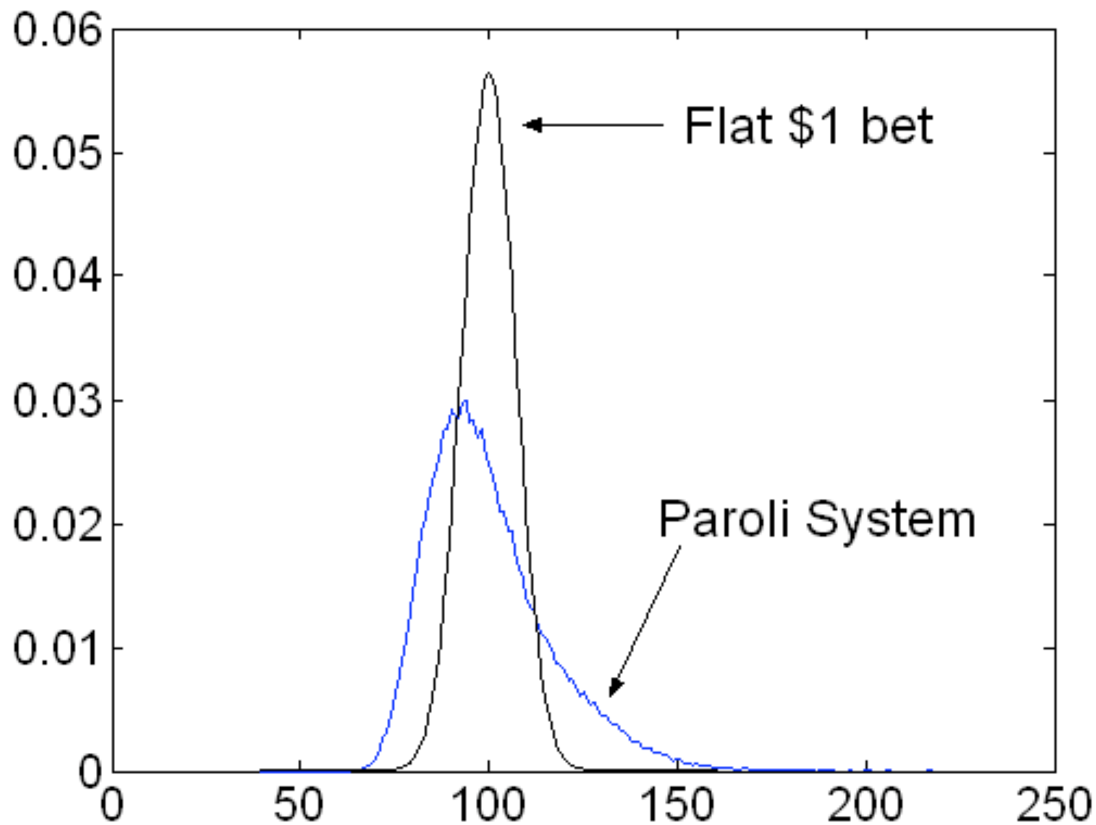
Betting systems CAN skew the general shape of the distribution of outcomes so long as the total expected winnings remains unchanged.

Two major classifications of betting systems:

- **Positive Progression**: Bet more as you win more. Makes it more likely to lose a little, but increases the chance of winning a lot.
- **Negative Progression**: Bet more as you lose more. Makes it more likely to win a little, but increases the chance of losing a lot.

Paroli System (Positive Progression)

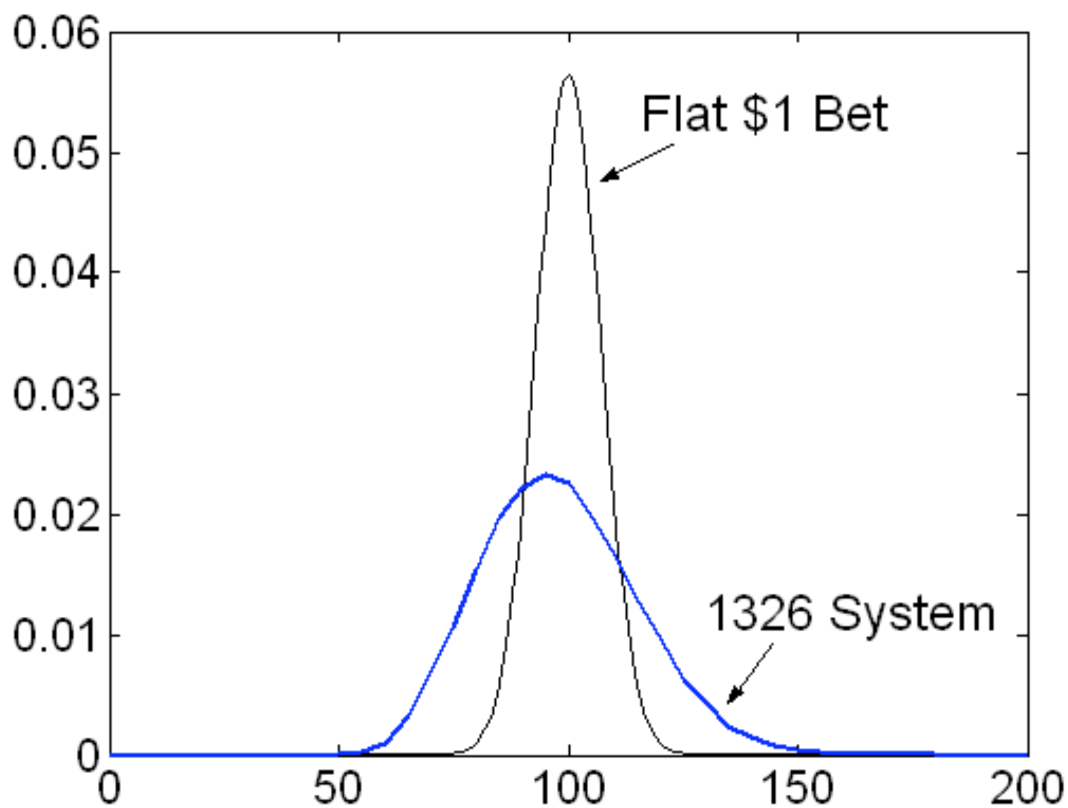
- Start with $100 and bet $1.
- After any win, increase last bet by $1 up to a max of $8.
- After any loss, next bet is $1.



Simulation: 100000 sets of 50 plays. No house edge.
Result: 15% more likely to lose.

1-3-2-6 System (Positive Progression)

- Start with $100 and bet $1.
- After any win, choose the next bet from the progression 1-3-2-6.
- After any loss, next bet is $1.

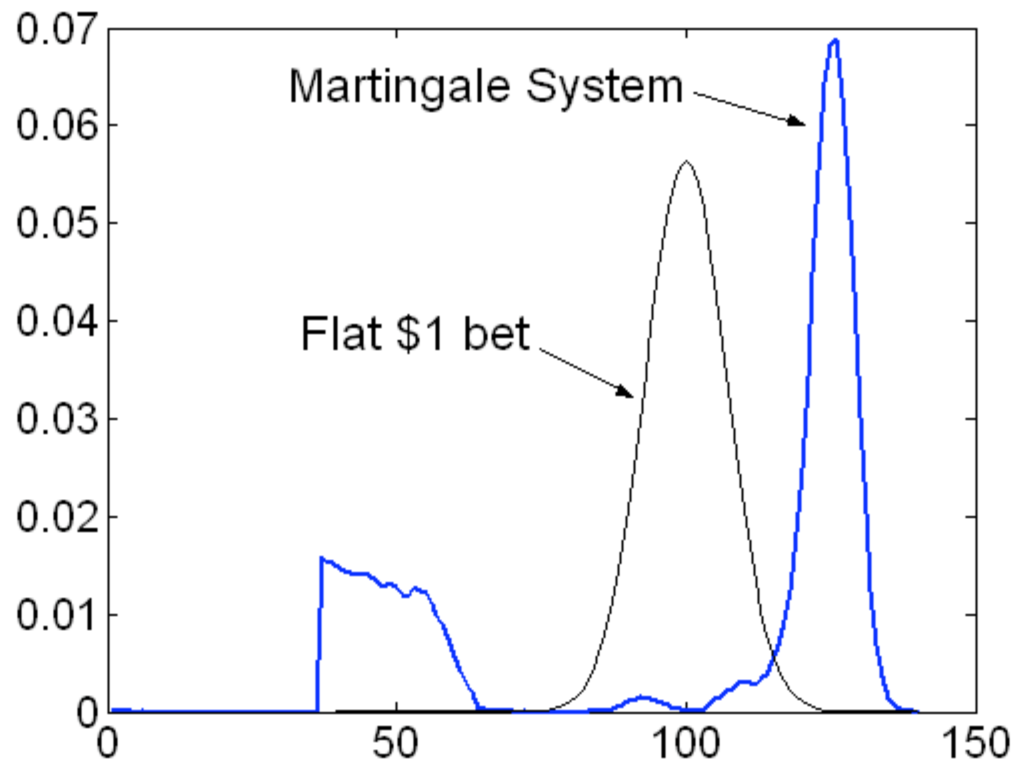


Simulation: 100000 sets of 50 plays. No house edge.
Result: 4% more likely to lose.

Martingale System

(Negative Progression)

- Start with $100 and bet $1.
- After any loss, double the next bet.
- After any win, next bet is $1.

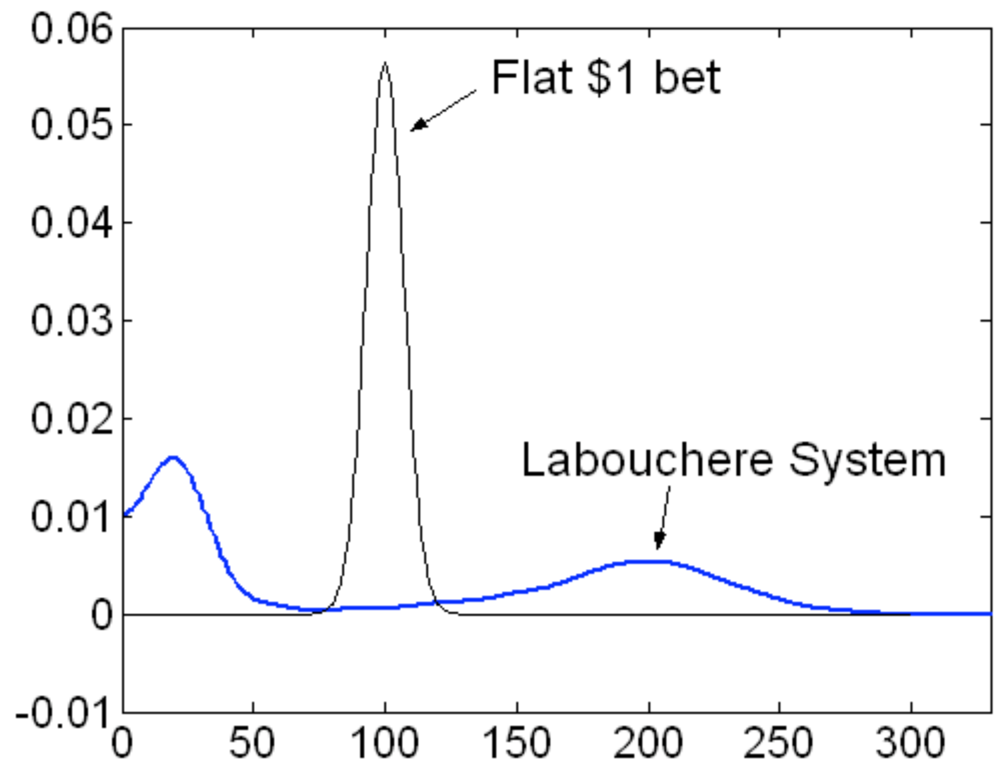


Simulation: 150000 sets of 50 plays (or until bet>money). No house edge. Result: 35% more likely to win.

Labouchere System

(Pseudo-Negative Progression)

- Start with $100 and the list {1 2 3 4 5 6}.
- Always bet the sum of the two list extremes.
- If win, cross them off. If lose, put the last bet at end of list.
- When all are crossed off, start over.
- If list has only one number, bet it.

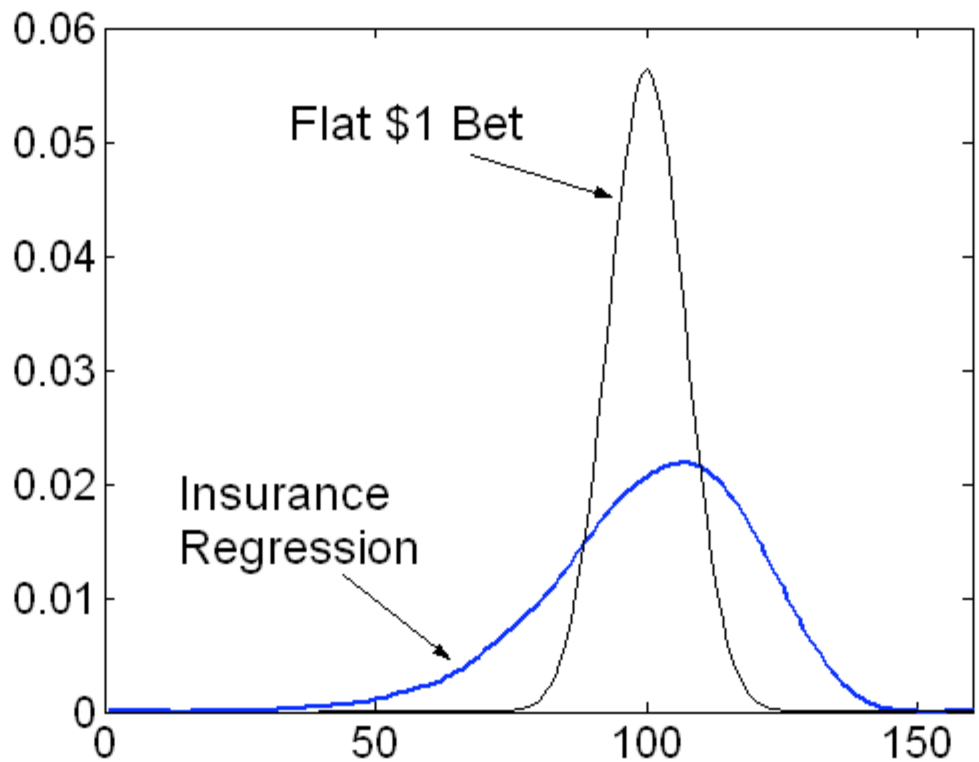


Simulation: 150000 sets of 50 plays. No house edge. Result: 8% more likely to lose.

Insurance Regression

Bet less as you win more.

- Start with $100 and bet $5.
- For every win, bet $.25 less than before.



Simulation: 100000 sets of 50 plays. No house edge.
Result: 8% more likely to win.

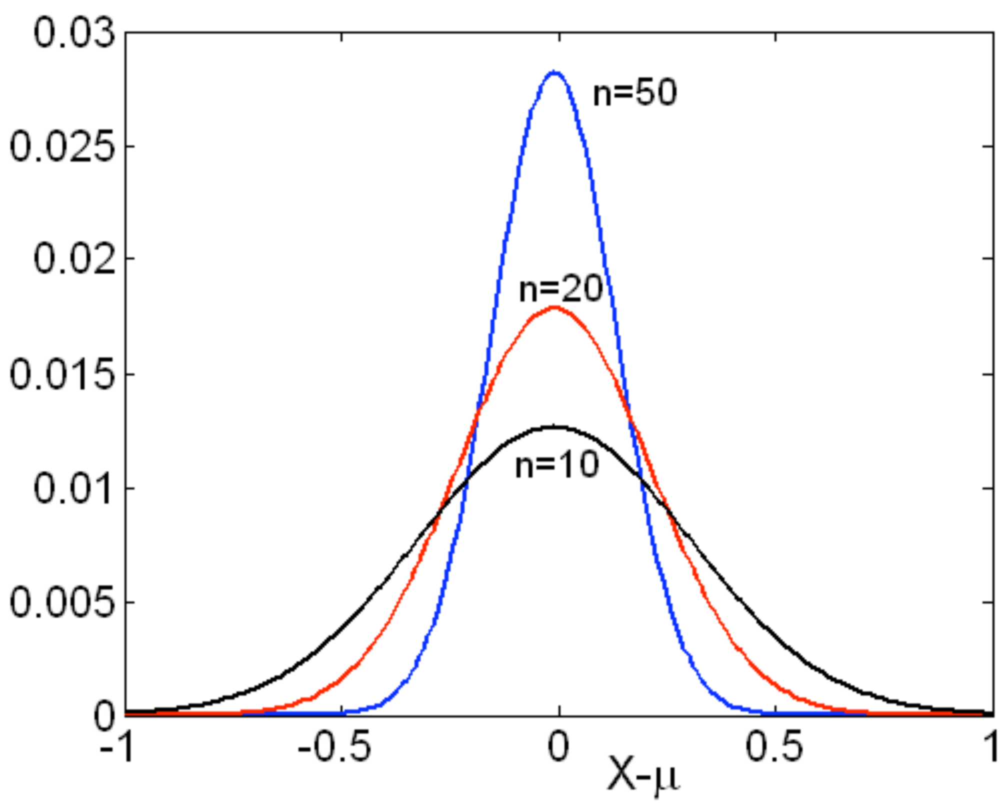
Quick quiz:

Suppose I want to gamble exactly $100 on a game with a house edge (expected gain<0). Would I lose more on average by playing small bets for a long time, or by betting it all at once?

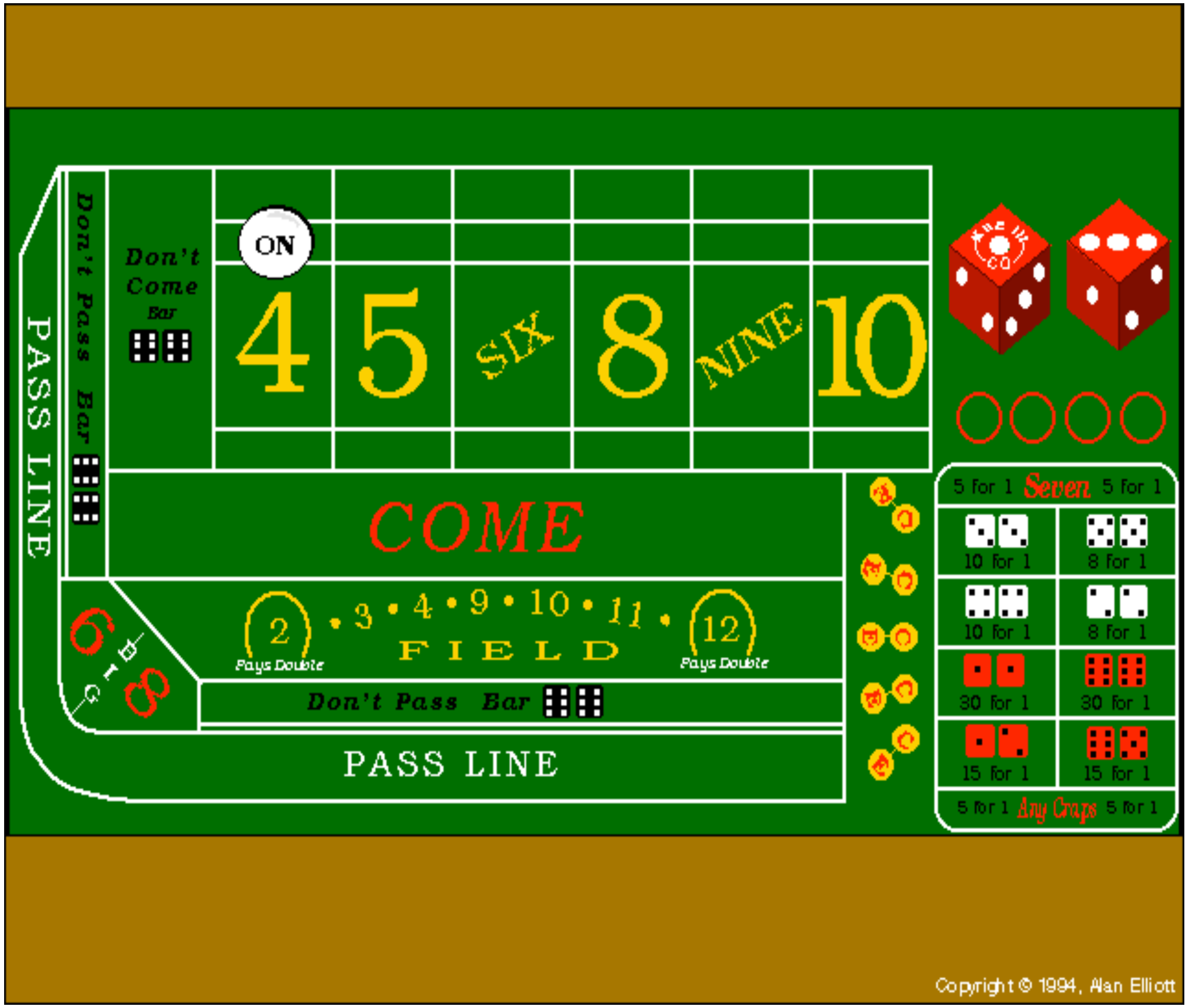
Other Ways to Customize

Suppose we have M dollars and wish to gamble all of it on n plays of a game with expected gain μ using flat bets of M/n dollars. Let X=winnings/M.

$$P_n(X) \propto \exp(-(X-\mu)^2 n/2)$$



Craps



(Pretty flashy, eh)

$$\underbrace{\frac{6}{36}+\frac{2}{36}=\frac{8}{36}}$$

$$P(\text{Pass}) = P(\text{Natural}) + P(\text{Shooter makes point}) = .4929$$

$$\overbrace{\frac{10}{36}\sum_{n=0}^{\infty}\left(\frac{25}{36}\right)^n\frac{5}{36}+\frac{8}{36}\sum_{n=0}^{\infty}\left(\frac{26}{36}\right)^n\frac{4}{36}+\frac{6}{36}\sum_{n=0}^{\infty}\left(\frac{27}{36}\right)^n\frac{3}{36}}$$

Point 6 or 8

Avoid 7 or point for n rolls

Point

$$\rightarrow \text{Expected Gain} = .4929-(1-.4929) = -.0141$$

1.41% house edge on pass-line

Laying Odds

If you bet on Pass or Don't Pass, you can *lay odds* or *back up* your bet once a point is established.

Shooter's point	Prob of winning	Payout on Pass Odds	Payout on Don't Pass Odds
6 or 8	5/11	6 to 5	5 to 6
5 or 9	2/5	3 to 2	2 to 3
4 or 10	1/3	2 to 1	1 to 2

$$\text{Expected Gain} = [\text{Payout Ratio}]P(\text{win}) - P(\text{lose}) = 0$$

NO HOUSE EDGE on ODDS!!! Laying odds is the best bet in the casino.

http://wizardofodds.com/craps/vegascrapsurvey.html

Odds Limits

Every casino decides the maximal odds bet allowed to a player in proportion to his Pass or Don't Pass bet.

$$\text{Max odds bet} = N \cdot (\text{Pass/Don't Pass Bet})$$

If betting maximum odds, the total bet has:

$$\text{Expected Gain} = -\frac{.0141}{1+N}$$

Go to casinos offering the largest N.

Craps Strategies

1) Minimize the house edge by choosing a casino with a high N value. Then play craps laying maximal odds bets.

2) Increase your chances of walking away with profit (without flirting much with disaster) by:
   - Using an Insurance Regression system.
   - Betting in smaller increments.

3) Increase chances of walking away with a big win by:
   - Using a Positive Progression system.
   - Concentrating your money into fewer bets.

Roulette


‘American Roulette’ table
00
0
1st 12
2nd 12
3rd 12
1 to 18
EVEN
ODD
19 to 36
2 to 1
2 to 1
2 to 1
2 0 28

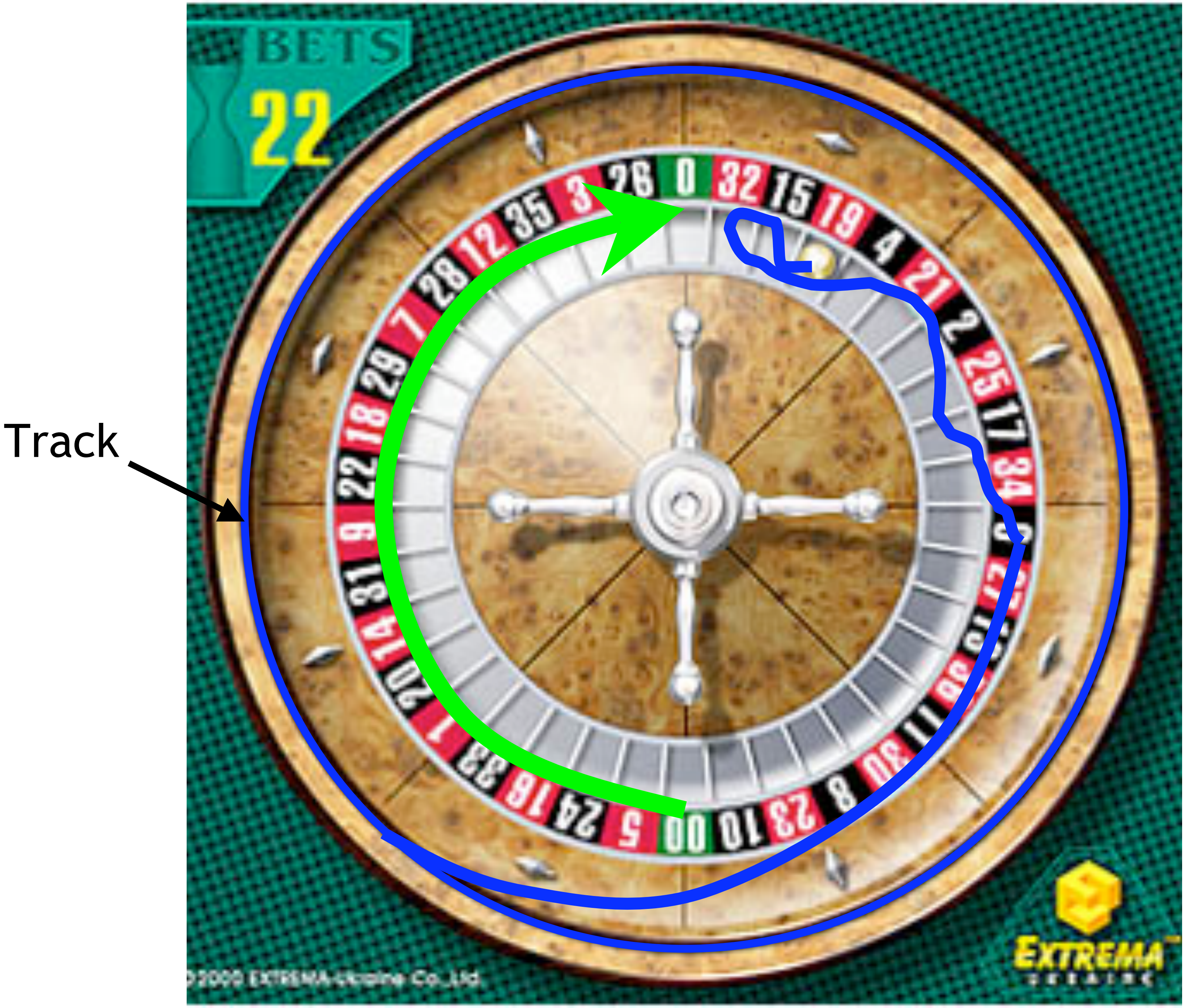
Bets/Probabilities/Gains

	Prob of Winning	Payout Ratio	Expected Gain
"5 number bet"	5/38	6 to 1	-7.89%
Any other bet (on n numbers)	n/38	(35/n) to 1	-5.23%

Large house edge.

If only we knew which half of the wheel the ball would land in…

Then: Expected Gain = (35/1)*(1/19)-18/19= 89.5% (!)


Track
BETS
22
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History of Beating the Wheel

- Joseph Jagger (1875): "The man who broke the bank at Monte Carlo". Exploited biases in one particular Roulette wheel.
- Claude Shannon & Ed Thorp (1961): Invented a wearable computer to predict Roulette ball trajectories.
- The Eudaemons (1978): Beat Vegas in real time using a shoe computer. Raised expected gain to +0.44.
- Gonzalo Garcia-Pelayo (1990): Won $1.77 million over several years by exploiting biased Roulette wheels in Madrid.
- Ritz gang (2004): Won $2.4 million in 2 days apparently using a laser scanner built into a cell phone.

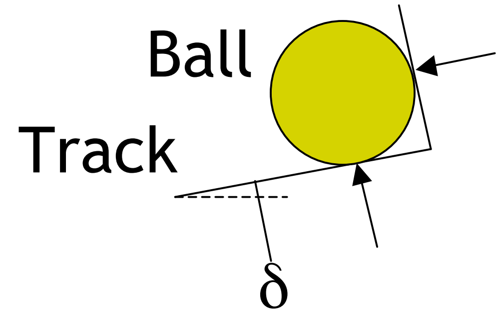
Roulette Physics (Sector Targetting)

The ball motion has three distinct phases:

1. Revolving around the track (~10 sec).
2. Traveling along the stator to the rotor (~1 sec).
3. Bouncing around the rotor (~2 sec).

The last phase is largely stochastic, but the other two have strong determinacies.

The Ball: Phase 1



Friction from bottom of track

$$\dot{\theta} = \Omega \qquad \dot{\Omega} = -\alpha\Omega^2 + \beta - \gamma \sin\theta$$

Angular deceleration

Forces from air drag and friction from track side wall

Correction term to account for tilted floors

For α, β, γ determinable constants.

$\alpha=\alpha(m,C_{drag},S,\rho_{air},\mu,\delta)$

$\beta=\beta(g,R,\mu,\delta)$

$\gamma=\gamma(g,R,\varepsilon)$

Closed Form

$$\frac{d\Omega}{dt} = \frac{d\Omega}{d\theta}\frac{d\theta}{dt} = \frac{1}{2}\frac{d}{dt}(\Omega^2) = -\alpha\Omega^2 + \beta - \gamma \sin\theta$$

$$\text{Let } \Gamma = \Omega^2. \quad \text{Then } \Gamma' + 2\alpha\Gamma = 2\beta - 2\gamma\sin\theta$$

$$\Rightarrow \Gamma = c_1 e^{-2\alpha\theta} + \frac{\beta}{\alpha} + \frac{2\gamma}{4\alpha^2+1}(\cos\theta - 2\alpha\sin\theta).$$

$$\boxed{\Omega^2 = c_1 e^{-2a\theta} + b^2 + \eta(\cos\theta - 2a\sin\theta)}$$

The Roulette Equation

Exit Condition

Ball exits track when the component of gravity pulling the ball inward equals the centrifugal force.

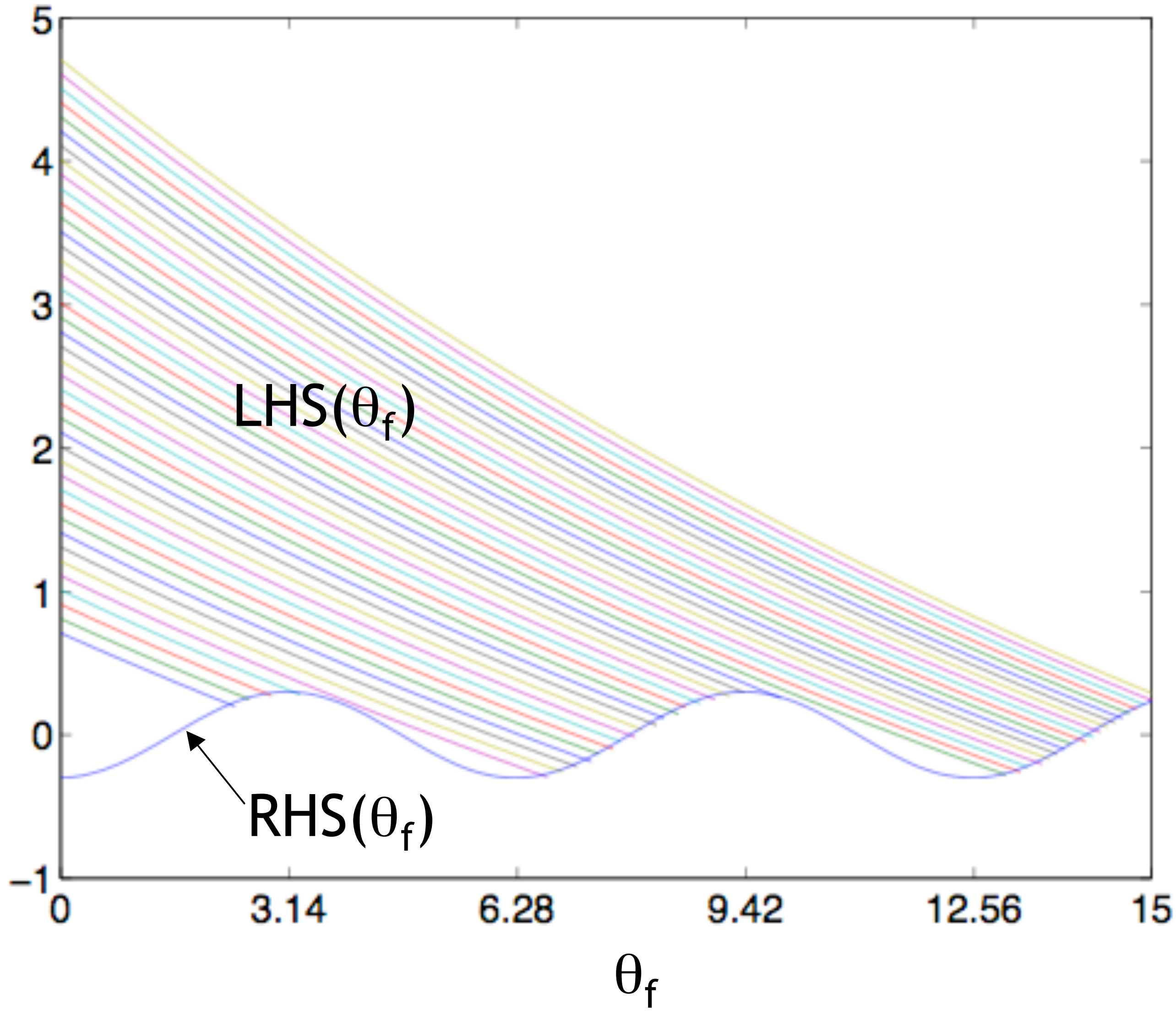
$$\Omega_f^2 = \frac{5g}{7R}(\tan\delta\cos\epsilon - \sin\epsilon\cos\theta_f)$$

$$\frac{5g}{7R}(\tan\delta\cos\epsilon - \sin\epsilon\cos\theta_f) = c_1 e^{-2a\theta_f} + b^2 + \eta(\cos\theta_f - 2a\sin\theta_f)$$

OR

$$\underbrace{c_1 e^{-2a\theta_f} - \frac{5}{7}\frac{g}{R}\tan\delta\cos\epsilon}_{\text{LHS}(\theta_f)} = \underbrace{\eta\left[(1 + \tfrac{1}{2}(4a^2+1))\cos\theta_f - 2a\sin\theta_f\right]}_{\text{RHS}(\theta_f)}$$

For tilted tables define $\theta=0$ at the low point.



5
4
3
2
1
0
−1
LHS(θf)
RHS(θf)
0
3.14
6.28
9.42
12.56
15
θf

Time of Exit

$$t_f = \int_0^{\theta_f} \frac{dt}{d\theta} d\theta = \int_0^{\theta_f} \frac{d\theta}{\sqrt{c_1 e^{-2a\theta} + b^2 + \eta(\cos\theta - 2a\sin\theta)}}$$

Trick to solving this: Let $\theta_f = 2\pi k + t_{extra}$

$$t_f = \int_0^{2\pi k} + \int_{2\pi k}^{2\pi k + t_{extra}}$$

$$\approx \boxed{\frac{1}{ab}\left[c_0 - \sinh^{-1}(e^{a\theta_f} \cdot \sinh c_0)\right] - \frac{\eta}{2}\frac{\sin\theta_f + 2a\cos\theta_f}{(c_1 e^{-2a\theta_f} + b^2)^{3/2}}}$$

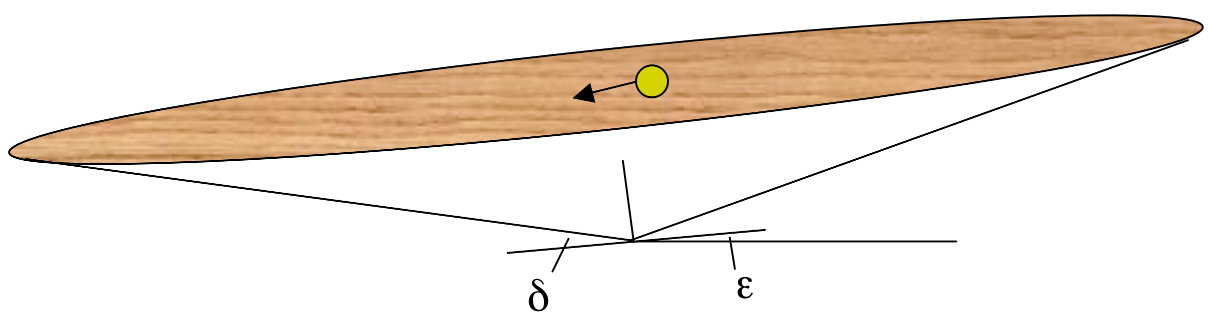
for $c_0 = -\sinh^{-1}\left(\frac{b}{\sqrt{c_1}}\right)$

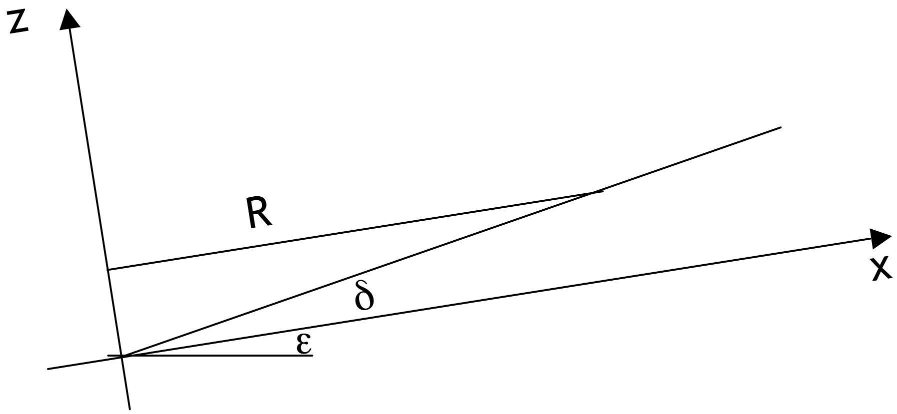
The Ball: Phase 2

We tacitly solve for ball's motion around the cone assuming it is not deflected by one of the metal diamonds.

Constrained motion ! use a Lagrangian.

i.e. Find an expression for kinetic energy T and potential energy U. Then the solution is the path that minimizes the integral of L=T-U between t=0 and t.





$$
\begin{aligned}
T = \text{Kinetic Energy} &= \frac{m}{2}v^2 + \frac{I}{2}\omega^2 \\
&= \frac{7m}{10}(\dot{x} + \dot{y} + \dot{z}) \\
U = \text{Potential Energy} &= mgh \\
&= mg(x \sin\epsilon + z\cos\epsilon)
\end{aligned}
$$

Now apply the constraint that the ball stays on the cone:
x=Rcosθ, y=Rsinθ, z=Rtanδ.
With this, we can express the Lagrangian as $L(R, \theta, \dot{R}, \dot{\theta})$.

Plug L into the Euler-Lagrange equation to find the equations of motion of the ball:

$$\begin{aligned}\ddot{R}/\cos\delta &= R\dot{\theta}\cos\delta - \frac{5}{7}g(\sin\delta\cos\epsilon - \cos\delta\sin\epsilon\cos\theta) \\ R\ddot{\theta} &= -2\dot{R}\dot{\theta} - \frac{5}{7}g\sin\epsilon\sin\theta\end{aligned}$$

Solve these equations numerically with initial conditions:

$$R(0) = R_0,\ \dot{R}(0) = 0,\ \theta(0) = \theta_f,\ \dot{\theta}(0) = \Omega_f$$

The Rotor

- Rotor spins on a shaft and is supported by ball bearings.
- At typical rolling speeds, the ball bearings drag the rotor according to a friction law. Thus,

$$I\dot{\omega} = \tau = const$$

- Therefore, angular deceleration is constant and the rotor angle follows an equation of the form:

$$\Theta(t) = \omega_0 t - \frac{k}{2}t^2$$

The Ball: Phase 3

This phase is highly non-deterministic.

Let v_2, θ_2, t_2 be the velocity, angular location, and time when the ball is at $R=R_{rotor}$ (from the end of Phase 2). Let ω_2 be the angular speed of the rotor at t_2.

Best guess:

$$< \theta_{bounce} >= q \cdot (v_2 + R_{rotor}\omega_2)$$

For $<\theta_{bounce}>$ is the mean forward angle the ball skips through during Phase 3 and q some constant.

Altogether

- Extracted Constants:
  - Roulette equation constants: a, b, η
  - For exit condition: R
  - Rotor constant: k
  - Skip distance: q
- What you must measure:
  - Initial Ω of the ball at a fixed point on the track (choose the lowest point if there is tilt)
  - ω of the rotor measured when the ball initially crosses the fixed point on track
  - Which number Z on the rotor is radially aligned with the fixed point on track when the ball first passes the fixed point.

USE ROTOR EQUATION TO DETERMINE $\Theta(t_2)$.
BET ON THE NUMBER CORRESPONDING TO THE ANGLE

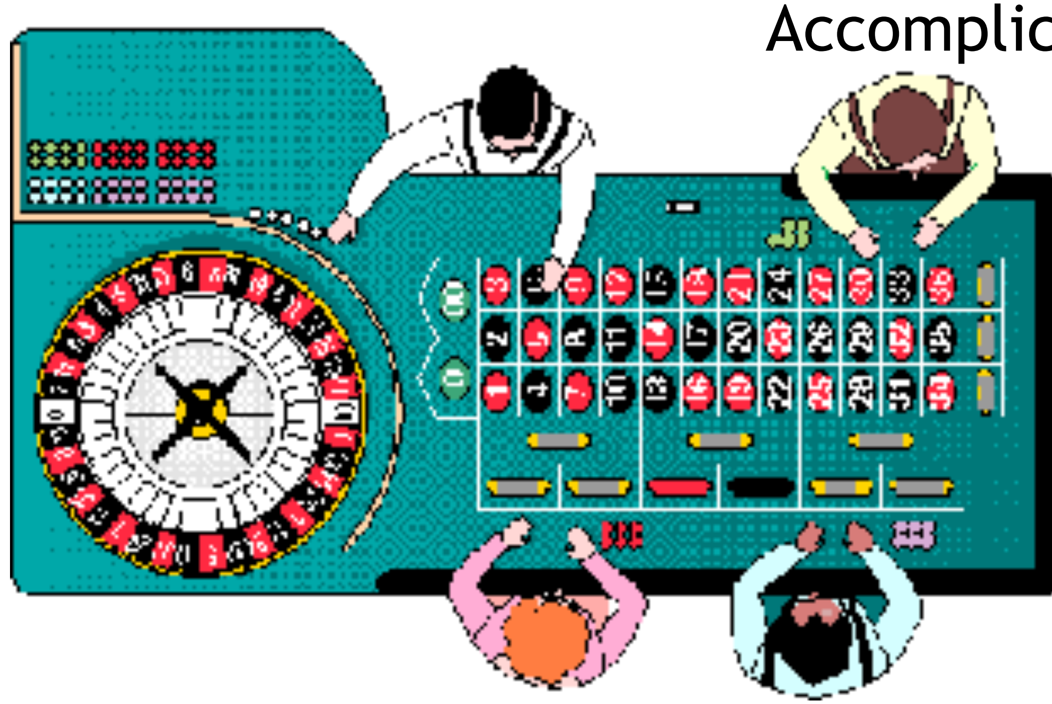
$$\theta_2 + \Theta(t_2) + < \theta_{bounce} >$$

BEYOND THE INITIAL NUMBER Z!!!

The Eudaemons

The Observer
Shoe

Accomplice



Observer

Conclusions

1. Betting systems and bet spreading can customize results to your personal psychological well-being.
2. If you're gonna play fair, play craps.
3. Don't expect to win unless you're wearing a computer.