2019 SPUR Conference

August 2, 2019

MIT, Room 2-449

9:20 am: Conference Opening by SPUR faculty advisors Prof. David Jerison and Prof. Ankur Moitra

9:30 am: Kaarel Haenni, "Wilf-equivalence in Weyl Groups and Signed Permutations" (mentor Yibo Gao)

10:00 am: Andrew Gu, "Arithmetic of Weighted Catalan Numbers" (mentor Yibo Gao)

10:30 am: Alexandra Hoey and Wanyi Xiao, "A Combinatorial Proof for a Generalized Reciprocity Theorem" (mentor Pakawut Jiradilok)

11:00-11:10 am: break

11:10 am: Qiuyu Ren, "On the Union of Essentially Distinct δ-tubes" (mentor Yuqiu Fu)

11:40 am: Douglas Stryker, "Construction of High Codimension Ancient Mean Curvature Flows and Codimension Bounds by the Tangent Flow at $-\infty$" (mentor Ao Sun)

12:10 pm: Yuan Yao and Shengtong Zhang, "Equiangular Lines with a Fixed Angle" (mentor Jonathan Tidor)

12:40-1:40 pm: lunch break

1:40 pm: Miles Johnson and Natalie Stewart, "Some Graphical Realizations of Two-Row Specht Modules of Iwahori-Hecke Algebras of the Symmetric Group" (mentor Oron Propp)

2:15 pm: Nelson Niu and Lillian Zhang, "A Multiplicity-free Macdonald Identity" (mentor Gurbir Dhillon)

2:45-2:55 pm: break

2:55 pm: Christopher Xu, "Traces of CM Values of Maass Forms" (mentor Yongyi Chen)

3:25 pm: Kevin Beuchot Castellanos, "Lifting to $\mathbb{Z}/p^2\mathbb{Z}$ and Splittings of the de Rham Complex" (mentor Robert Burklund)

3:50 pm: Conference Closing

Kaarel Haenni

Wilf-equivalence in Weyl Groups and Signed Permutations

Mentor: Yibo Gao

Project suggested by Prof. Alexander Postnikov

In the first part of the paper, we consider Wilf equivalence for root systems. To that end, we find all subspace root systems in root systems of classical types. We then give an explicit description of the restriction map from the Weyl group of the root system to the Weyl group of the subspace root system. We provide a reduction of root system pattern avoidance to avoiding a set of usual permutation patterns in a permutation or avoiding a set of signed patterns in a signed permutation. We find that for patterns of type A_k for $k \leq 4$, there are no nontrivial root system Wilf equivalences. In the second part of the paper, we consider a different notion of permutation patterns in signed permutations. Anderson and Fulton have conjectured that 1234 and 2143 are Wilf equivalent for signed permutations. As a potential step towards settling this conjecture, we provide generating trees for 1234 and 2143-avoiding signed permutations.

Andrew Gu

Arithmetic of Weighted Catalan Numbers

Mentor: Yibo Gao

Project suggested by Prof. Alexander Postnikov

In this paper, we study arithmetic properties of weighted Catalan numbers. In a previous paper, Postnikov and Sagan found conditions under which the 2-adic valuation of the weighted Catalan numbers are equal to the 2-adic valutation of the unweighted Catalan numbers. We prove the same result under weaker conditions by considering a mapping from a class of functions to 2-adic integers. These methods are also extended to q-weighted Catalan numbers. Finally, we prove some results on the periodicity of weighted Catalan numbers modulo an integer and apply them to the specific case on the number of Morse links.

Alexandra Hoey and Wanyi Xiao

A Combinatorial Proof for a Generalized Reciprocity Theorem

Mentor: Pakawut Jiradilok

Project suggested by Prof. Alexander Postnikov

We study "proper pairings" for finite simple graphs. These are combinatorial objects that Huang and Postnikov used to give a bijective proof of Pak and Postnikov's reciprocity formula for the spanning forest polynomial f_G. We find that by introducing the "component graph" – a combinatorial object related to proper pairings – we are able to see new enumerative properties for these graph objects. As a result of our study, we give another combinatorial proof of the reciprocity theorem similar to Huang and Postnikov's. Furthermore, we generalize f_G to a polynomial $f_{G,H}$ that records the spanning trees of a graph that contain a fixed subgraph, and we show that these generalized polynomials exhibit a similar reciprocity property.

SESSION 2

Qiuyu Ren

On the Union of Essentially Distinct δ-tubes

Mentor: Yuqiu Fu

Project suggested by Prof. Larry Guth

We say two δ-tubes (dimension $\delta \times \cdots \times \delta \times 1$) in $\mathbb{R}^n$ are essentially distinct if the measure of their intersection is smaller than a half of a single δ-tube. For a collection of essentially distinct δ-tubes, we give the asymptotically sharp lower bound for the measure of their union. Then we characterize all sharp examples. We will give a new measurement of convexity based on the X-ray transform.

Douglas Stryker

Construction of High Codimension Ancient Mean Curvature Flows and Codimension Bounds by the Tangent Flow at $-\infty$

Mentor: Ao Sun

Project suggested by Prof. William Minicozzi

We construct a class of compact ancient solutions to the mean curvature flow in high codimension. Using these examples, we provide novel insight about the recent entropy-based codimension bound for ancient solutions proved by Colding and Minicozzi. Inspired by the limiting behavior of these examples as $t \to -\infty$, we prove a sharp codimension bound for compact ancient solutions that converge sufficiently rapidly to homothetic shrinking solutions as $t \to -\infty$.

Yuan Yao and Shengtong Zhang

Equiangular Lines with a Fixed Angle

Mentor: Jonathan Tidor

Project suggested by Dr. Zilin Jiang and Prof. Yufei Zhao

Let $N_\alpha(d)$ be the maximum number of lines in $\mathbb{R}^d$ through the origin that pairwise intersect at a fixed angle $\arccos \alpha$. In this paper, we completely resolve the long-standing problem of determining $\lim_{d\to\infty} N_\alpha(d)/d$ for every $\alpha \in (0, 1)$. Furthermore, we determine the exact value of $N_\alpha(d)$ for large d when the limit is not 1.

Miles Johnson and Natalie Stewart

Some Graphical Realizations of Two-Row Specht Modules of Iwahori-Hecke Algebras of the Symmetric Group

Mentor: Oron Propp

Project suggested by Prof. Roman Bezrukavnikov

We consider the Iwahori–Hecke algebra of the symmetric group on $2n + r$ letters with parameter q. Let e be the smallest positive integer such that the q-number $[e]_q = 0$, or set $e = \infty$ if none exist. We modify Khovanov's crossingless matchings to include $2n$ "nodes" and r "anchors," and prove in the case $e > n+r+1$ that the associated module is isomorphic to the Specht module $S^{(n+r,n)}$ which corresponds to the partition $(n+r, n) \vdash 2n+r$. We then give heuristics in support of the general case, including explicit composition series for $e = n+r+1$ and for $2n+r \leq 6$. Lastly, when $e = 5$, we prove an isomorphism between the irreducible quotient $D^{(n+r,n)}$ with $r \leq 3$ and some subrepresentations of Jordan–Shor's Fibonacci representation. We provide explicit transition matrices between this representation and the crossingless matchings representation for $2n + r \leq 6$.

Nelson Niu and Lillian Zhang

A Multiplicity-free Macdonald Identity

Mentor: Gurbir Dhillon

Project suggested by Gurbir Dhillon

Given an irreducible (untwisted, reduced) affine root system, let Φ be the set of affine roots and let W be the associated affine Weyl group. Pick a set of simple roots $\Sigma \subseteq \Phi$, and let $\delta \in \Phi$ be the unique indecomposable positive imaginary root. We show the following equality of formal characters:

$$\sum_{w \in W} \prod_{\alpha \in \Sigma} \frac{1}{1 - e^{-w\alpha}} = \frac{1}{1 - e^{-\delta}}.$$

The existence of such an formula, which is a multiplicity-free version of a famous identity due to Macdonald, confirms a conjecture of Dhillon–Khare (2018).

To obtain this formula, we prove the following multivariate Macdonald identity. Let Φ_m be the multiset of all roots, real and imaginary, in which each root appears with its standard multiplicity. Let Φ_m^+ (resp. Φ_m^-) be the submultiset of every instance of every positive (resp. negative) root in Φ_m. For an arbitrary submultiset B of Φ_m^+, let B^- denote the image of B after each positive root is sent to its corresponding negative root, and let $|B|$ be the sum of the roots in B. Finally, let $\Phi_m(w, B) = \Phi_m^+ \cap w(B \cup \Phi_m^- \setminus B^-)$. We show the following multivariate Macdonald identity for indeterminates u_α for each $\alpha \in \Phi_m^+$:

$$\sum_{w \in W} \prod_{\alpha \in \Phi_m^+} \frac{1 - u_\alpha e^{-w\alpha}}{1 - e^{-w\alpha}} = \sum_{n=0}^{\infty} e^{-n\delta} \sum_{w \in W} \sum_{\substack{B \subseteq \Phi_m^+, \\ |B| = n\delta}} (-1)^{\#B} \prod_{\alpha \in \Phi_m(w,B)} u_\alpha.$$

A specialization of each u_α to integer values recovers the first formula.

Christopher Xu

Traces of CM Values of Maass Forms

Mentor: Yongyi Chen

Project suggested by Prof. Ken Ono (Emory University)

For an automorphic form f, the *trace generating series* of f is a Fourier expansion whose coefficient of degree D is the sum of the values of f at imaginary quadratic integers of discriminant D. Bruinier and Funke show that when f is a modular function, the trace generating series appears in the positive exponents of the *theta lift* of f, a weight $3/2$ nonholomorphic modular form. Building off of their work, we give an analogous formula for the theta lift of f containing the trace generating series when f is a nonholomorphic weight 0 Maass form.

Kevin Beuchot Castellanos

Lifting to $\mathbb{Z}/p^2\mathbb{Z}$ and Splittings of the de Rham Complex

Mentor: Robert Burklund

Project suggested by Robert Burklund

In this paper we work with a scheme in characteristic p and study the relationship between the existence of a smooth lifting to $\mathbb{Z}/p^2\mathbb{Z}$ and the de Rham complex splitting into a direct sum of its cohomology sheaves. Deligne and Illusie proved that when the scheme has a smooth lifting and the dimension is less than p such a splitting exists. There are known examples of schemes that do not admit a lifting and for which the de Rham complex does not split. Nevertheless, there are no known examples of schemes admitting a lifting whose de Rham complex does not split. In this paper we look at schemes of higher dimensions admitting a lifting to $\mathbb{Z}/p^2\mathbb{Z}$ and measure how close the de Rham complex is to splitting by studying the Hodge-de Rham spectral sequence.