Jiwon Choi

Geometric Complexity of Planar Drawings

under the direction of Alexey Balitskiy

Abstract

We say that a planar drawing of a graph is 1-thick if the distance between the images of any two vertices, a vertex and an edge, and two non-adjacent edges is at least 1. We prove that the cylinder mesh graph $C_{M,N}$ has a 1-thick drawing inside a ball of radius $C \cdot (\sqrt{MN} + N)$ for some absolute constant C. Moreover, we prove that the value $\sqrt{MN} + N$ is sharp up to a constant factor.

Sean Elliott

Modeling Networks of Evolving Populations

under the direction of Dominic Skinner

Abstract

We study the Fisher-Eigen process, a model for evolution. We simulate this process over a 2-dimensional energy landscape using MATLAB. We use this simulation to construct a simpler model by representing the landscape as a network, where each node represents a group of organisms with similar genes. We then create a model of ordinary differential equations over this network. Our model finds the correct equilibrium solution. Our model can be used to simplify modeling of high-dimensional data and to predict behavior based on data taken at a single time.

Karen Ge

Generating the Coefficient Field of Newforms with Inner Twists

under the direction of Robert Burklund

Abstract

We investigate modular forms, which are periodic functions that are related to elliptic curves. Modular forms are periodic, and so have a Fourier expansion. We look at the field E_f generated by adjoining these Fourier coefficients to $\mathbb{Q}$. When this field has so-called inner twists, a property of the automorphisms of E_f, we demonstrate an explicit connection between the odds of randomly generating the group of these twists and randomly generating the coefficient field E_f. Previous work focused on the probability that a single prime could generate the whole field; we extend these results by investigating the probability that n primes generate this field. We first relate previous results more directly to group theory by proving an explicit formula, then we show that this formula can be extended to choosing n primes and describe the group theoretic analogue.

Kayson Hansen

The Modular Representation Theory of Cyclic Groups of Prime Power Order

under the direction of Hood Chatham

Abstract

It is well-known that representations of a group G over a field K are in bijection with modules over the group algebra $K[G]$—this is the basis for the field of modular representation theory. We study the modular representation theory of cyclic groups with prime power order C_{p^k} over finite fields $\mathbb{F}_p$. There is a broad background in the literature on the representation theory of C_{p^k} over a finite field $\mathbb{F}_n$ when $n \nmid p^k$, but little is known when $n \mid p^k$, which is the case we study. We find a basis for the representation ring of $\mathbb{F}_p[C_{p^k}]$, which allows us to find a much simpler structure to describe the representation ring. Our results help us better understand the representation theory of cyclic groups, which have applications in Number Theory and the Langlands Program.

Kaiying Hou

Strang Splitting for the Variable-coefficient Burgers Equation

under the direction of Ruoxuan Yang

Abstract

In this paper, we use Strang splitting to solve the variable-coefficient Burgers' equation $u_t = a(t)u_{xx} + b(x,t)uu_x$. We prove that if the equation is well-posed, then the Strang splitting method has first-order convergence by showing the stability and consistency of Strang splitting. The convergence of Strang splitting allows us to numerically solve the variable-coefficient Burgers' equation by solving its two constituent equations: the heat equation and the inviscid Burgers' equation. We have good numerical methods for these two equations respectively.

Aknazar Kazhymurat

Topological uniqueness results for Lefschetz fibrations over the disc

under the direction of Bar Kartal

Abstract

We prove that a Lefschetz fibration over the disc that, after compactification, has the same singular fibers as an extremal rational elliptic surface can be obtained by deleting a singular fiber and a section from the rational extremal elliptic surface, i.e. such a Lefschetz fibration is determined up to topological equivalence by its set of singular of fibers. In general, a Lefschetz fibration is not determined by its set of singular fibers.

The main theorem is the first known topological uniqueness result for Lefschetz fibrations of genus 1 over the disc (as opposed to the sphere). We get a complete clasification of Lefschetz fibrations with 2 I_1 fibers as a byproduct of our results.

The proof is inspired by homological mirror symmetry and Karpov–Nogin's theorem on constructivity of helices on del Pezzo surfaces.

It would be interesting to extend our results to the case of Lefschetz fibrations that have the same singular fibers as an extremal K3 surface.

Chavdar Lalov

Cyclic Koszul algebras and oriented graphs

under the direction of Guangyi Yue

Abstract

Koszul algebras are one of the most studied types of quadratic algebras due to their numerous applications in other fields of mathematics. We generalise the notion of a Koszul algebra through the use of oriented graphs. The defining properties of Koszul algebras can be attached to a special type of oriented graphs: a line with vertices, where all edges are oriented towards the right. Reversing this combinatorial correspondence, we introduce a new type of algebras, Cyclic Koszul algebras, associated with oriented cycles. Firstly, we show that the new structure recovers some of the fundamental properties of classical Koszul algebras; for instance, if an algebra is Cyclic Koszul, then so is its dual algebra. Conversely, when $\dim V = 2$, we prove that the symmetric algebra $S(V)$ is not Cyclic Koszul, contrasting the fact that it is Koszul. However, we partially prove a conjecture that if we quantize the symmetric algebra $S(V)$, it will be Cyclic Koszul in most cases.

Kevin Liu

Number Fields Generated by Torsion Points on Elliptic Curves

under the direction of Chun Hong Lo

Abstract

Let E be an elliptic curve over $\mathbb{Q}$ and p be an odd prime. Assume that E does not have a p-adic point of order p, i.e. $E(\mathbb{Q}_p)[p] = 0$. For each positive integer n, define $K_n := \mathbb{Q}(E[p^n])$. Finding the class number of general number fields is a difficult problem in number theory, and we investigate the specific case of the class number of K_n. There is an injective homomorphism mapping $\mathrm{Gal}(K_n/\mathbb{Q})$ to $GL_2(\mathbb{Z}/p^n\mathbb{Z})$. We determine a lower bound on the order of the p-Sylow subgroup of the class group of K_n in terms of the Mordell-Weil rank of E in cases where this homomorphism is not necessarily surjective.

Roshan Warman

Generalization of Bridge Length to other Cartan-Killing Types

under the direction of Yibo Gao

Abstract

The positive Grassmannian has become increasingly studied since the discovery of its close connections with permutations. However, these results have been limited to characterizing only elements of type A Weyl groups. We present similar characterizations for the elements of the remaining types of Weyl groups ($B_n, C_n, D_n, E_6, E_7, E_8, G_2, F_4$) and explicit bounds for their lengths. Further we present the canonical intuition in terms of the Grassmannian and present new methods to consider the Grassmannian beyond type A.

Katie Wu

Microscopic Simulation of Growing Bacterial Swarms

under the direction of Dominic Skinner

Abstract

Systems of bacteria swimming in a single layer of fluid above a nutrient-rich substrate show exponential growth in area as the bacteria push the boundaries of the fluid outwards. Considering bacteria to be self-propelled particles making up an active matter system, this exponential growth can be explained by a nonlocal pressure driven by growth at the center of the swarm, though no theoretical basis for such a pressure has been found. We simulate bacterial multiplication and interactions in the presence of a movable wall using GPU code in order to investigate the forces on the edges of a bacterial swarm and the density of the bacteria as a function of position and time. We verify through the simulation that the front grows exponentially and find that the local bacterial density near the movable wall grows linearly.

Saba Zerefa

A Computational Approach To Intrinsic Linkedness in Complete Graphs

under the direction of Vishal Patil

Abstract

Connections among topology, geometry, and graph theory arise when analyzing structural components of nonplanar graphs. Initial discovery into intrinsic knottedness and linkedness of graphs began with proofs that K_6 is intrinsically linked and K_7 is intrinsically knotted. Previously it had been shown that K_9 is intrinsically linked in linear embeddings and K_{10} is intrinsically linked in all spatial embeddings with linking number ≥ 2. We show through a computational approach that K_7 is not intrinsically linked in linear embeddings with linking number ≥ 2, and that K_8 is not intrinsically linked in spatial embeddings with linking number ≥ 2. It is still unknown whether K_9 is intrinsically linked in spatial embeddings or K_8 is intrinsically linked in linear embeddings with linking number ≥ 2.