Graph Theory and Tesselations

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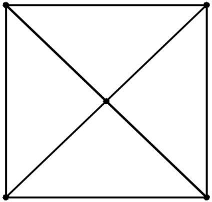
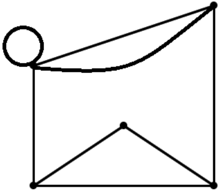
Introduction: Graphs

Definition of a graph

A *graph* $G = (V, E)$ is a set of vertices V together with a set of edges E connecting these vertices.



A simple graph

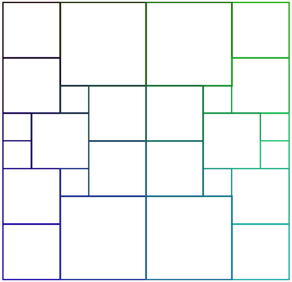


A non-simple graph

Introduction: Tilings

Definition of a tiling

Informally, a *tiling* (tessellation) is a collection P of geometric shapes with no overlap, and no empty space in between.
Specifically, a *square tiling* of a rectangle R is a set $T = (T_v, v \in V)$ of squares with disjoint interiors whose union is R.



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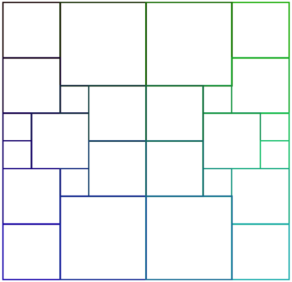


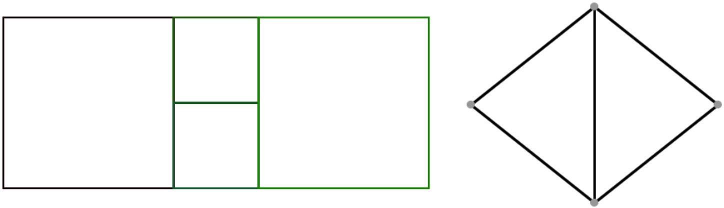
How does a tiling relate to a graph?

Contacts Graph

A *Contacts Graph* captures the combinatorics of a packing.

Contacts Graph

Consider tiling $T = (T_v : v \in V)$. The contacts graph of P is the graph $G = (V, E)$ where distinct vertices $v, w \in V$ are joined by an edge if and only if $T_v \cap T_w \neq \varnothing$.



A square tiling and its contact graph.

Connecting Tilings and Graphs

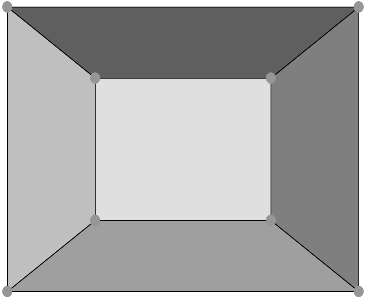
Brooks et. al: Square tiling where no two squares are equal;
Sets of squares correspond to vertices;
Put weights on edges.

Schramm: No restrictions on the squares;
Squares correspond to vertices;
Put weights on vertices.

Planar graph and its boundaries

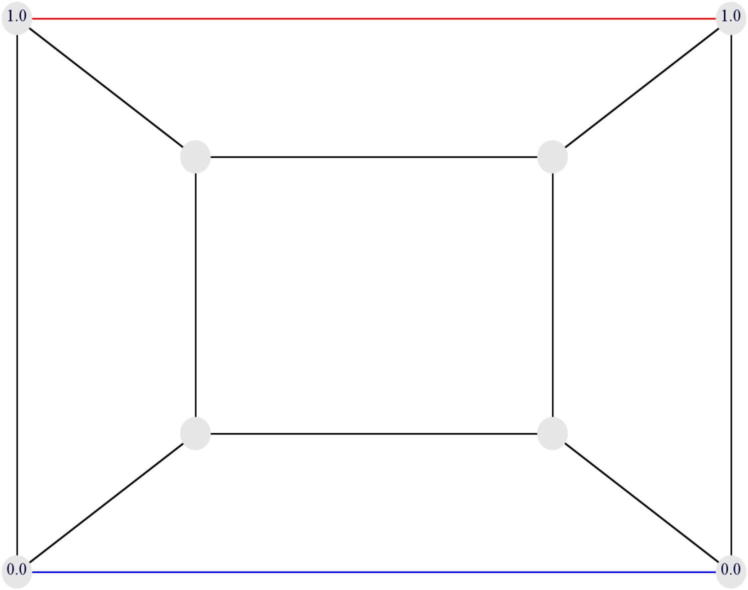
Planar Graph

A *Planar Graph* is a graph that can be embedded in the plane.
This embedding allows us to rigorously define *faces* of the graph.

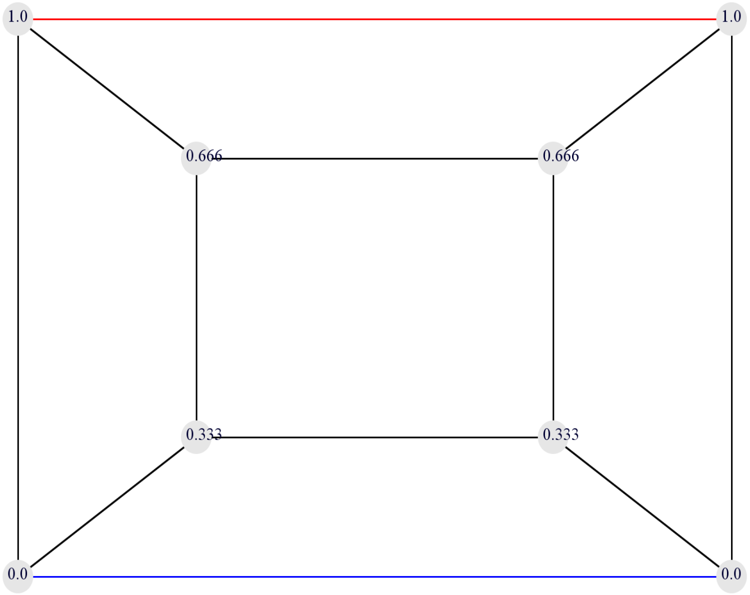


Planar graph with 6 faces.
One of these faces is unbounded.

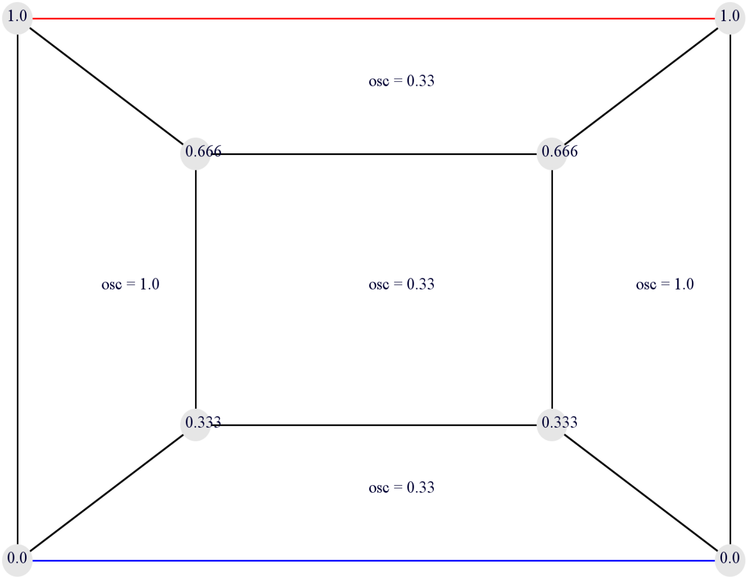
Our extremal Problem



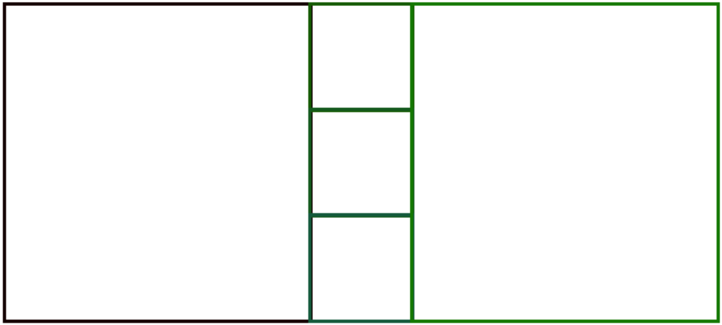
Our extremal Problem



Our extremal Problem

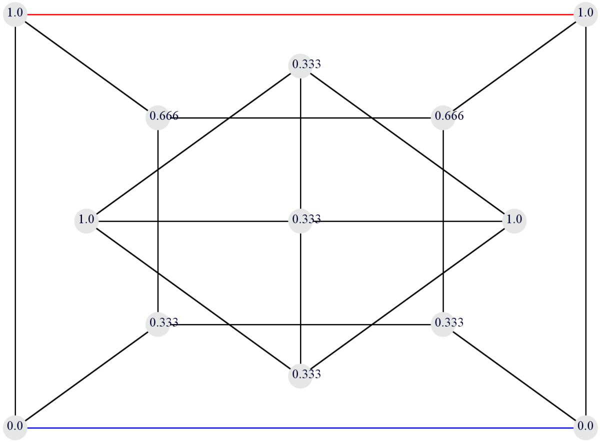


Our extremal Problem



Connections to previous work

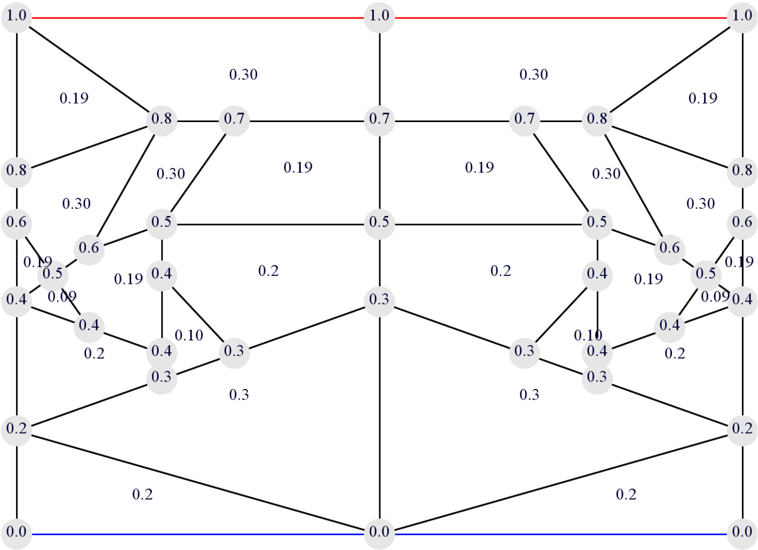
If we view our planar graph as a tiling, and consider its contact graph, our extremal problem becomes similar to Schramm's. Therefore, it is considered a *dual* of Schramm's problem.



Calculating extremal weights

We calculate extremal weights by first calculating the extremal oscillations.
Our algorithm *converges* to the extremal oscillations with error bound $E_n \leq O(n^{-\frac{1}{2}})$.
We then convert the extremal oscillations back into extremal weights on vertices.

Example



1.0
1.0
1.0
0.30
0.30
0.19
0.19
0.8
0.7
0.7
0.7
0.8
0.8
0.30
0.19
0.19
0.30
0.8
0.30
0.30
0.6
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0.5
0.5
0.6
0.6
0.6
0.19
0.5
0.19
0.4
0.2
0.2
0.4
0.19
0.5
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0.4
0.09
0.3
0.09
0.4
0.4
0.4
0.10
0.10
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0.2
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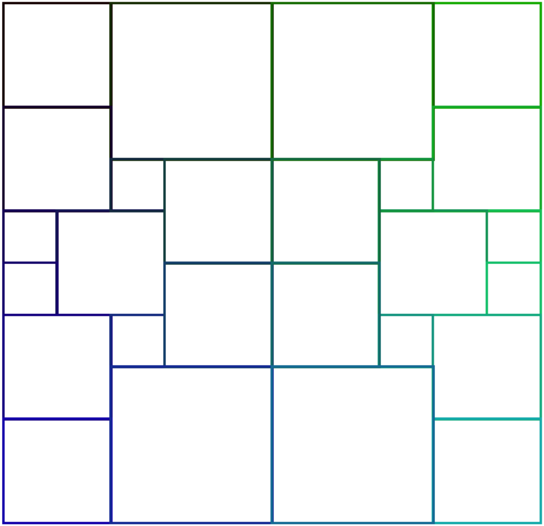
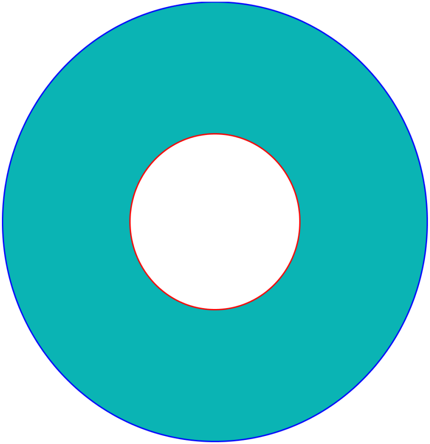
Example (cont.)



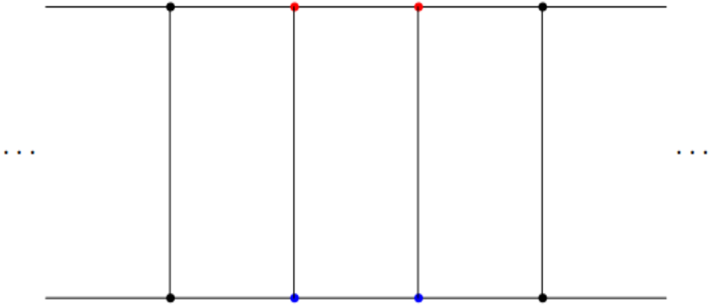
Future Work

Extending the configuration to an annulus:



Future Work (cont.)

Infinite graphs



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