Maximal Self-Intersection Number of Curves on Surfaces

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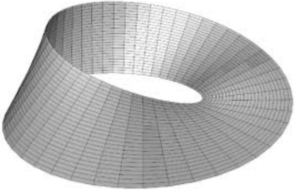
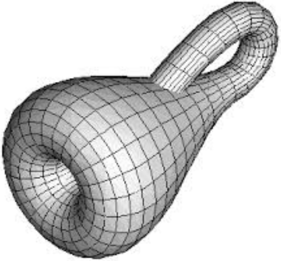
Surfaces

Surfaces

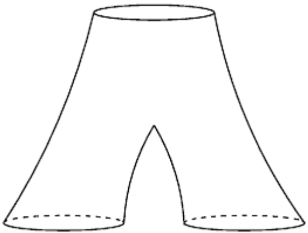


Mobius Strip

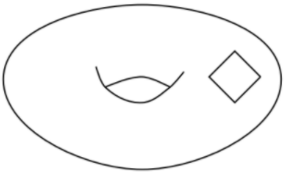


Klein Bottle

Surfaces

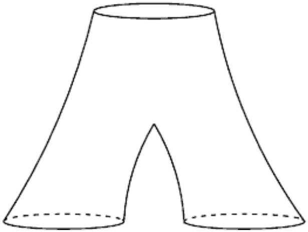


Pair of Pants

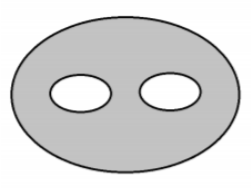


Torus with One Boundary

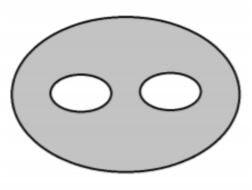
Surface Words



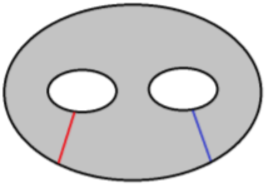
Pair of Pants



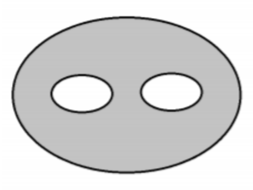
Surface Words



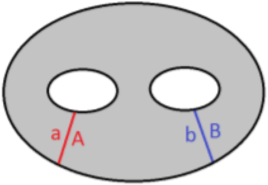
Still a Pair of Pants



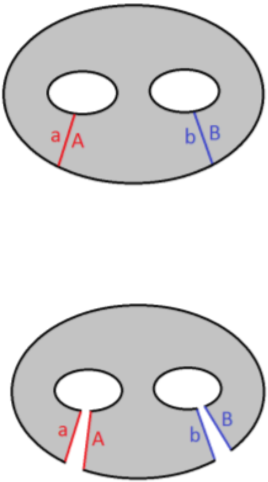
Surface Words



Still a Pair of Pants



Surface Words



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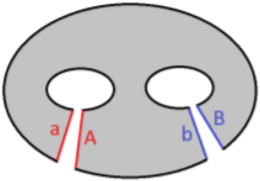

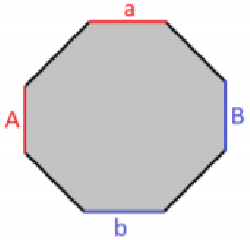

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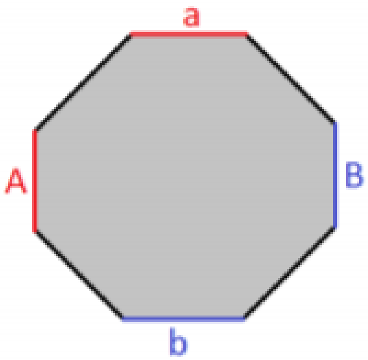
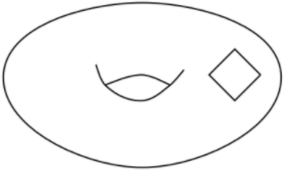
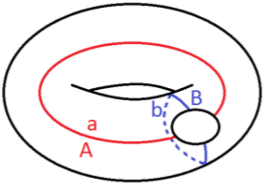


Surface Word: aAbB

Surface Words



Torus with One Boundary



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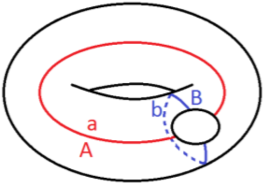
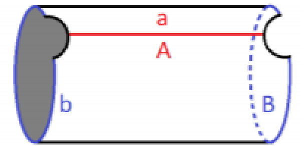
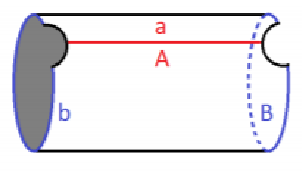
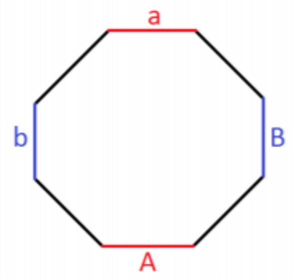


Still a Torus with One Boundary

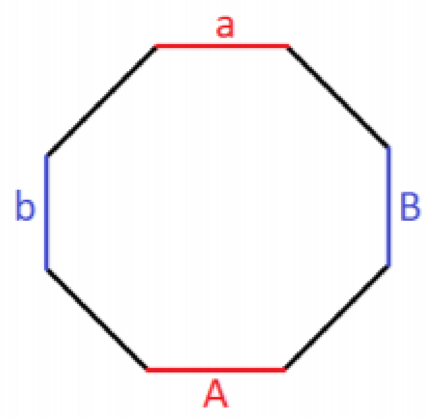


Surface Words



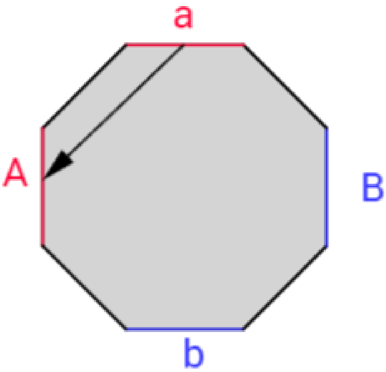


Surface Words



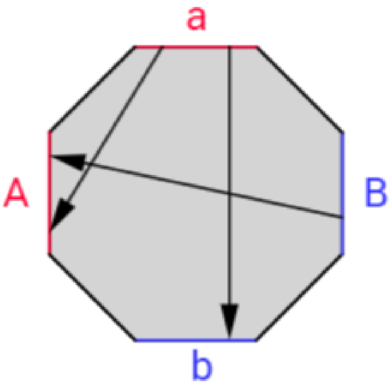
Surface Word: abAB

Curves on Surfaces



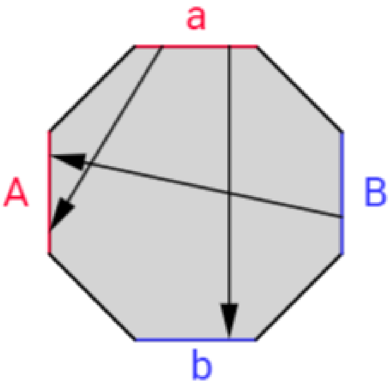
Curve Word: A

Curves on Surfaces



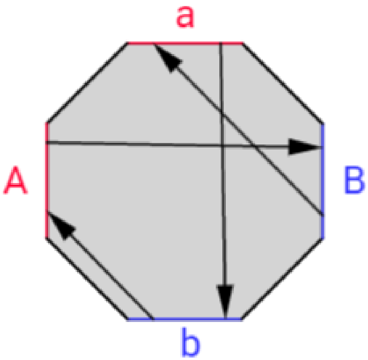
Curve Word: AbA

Curves on Surfaces



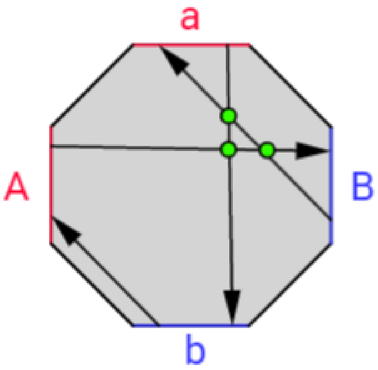
Curve Word: AbA
Word Length: 3

Curves on Surfaces



Curve Word: aBAb
Word Length: 4

Self-Intersections



Curve Word: aBAb
Word Length: 4
Self-Intersection Number: 3

Algorithm Output

Algorithm Output

L	maxSI(L) on aAbB	maxSI(L):L^2	maxSI(L) on abAB	maxSI(L):L^2
2	1	0.25	0	0
3	2	0.2222222222	0	0
4	3	0.1875	1	0.0625
5	6	0.24	2	0.08
6	7	0.1944444444	4	0.1111111111
7	12	0.2448979592	6	0.1224489796
8	15	0.234375	9	0.140625
9	20	0.2469135802	12	0.1481481481
10	23	0.23	16	0.16
11	30	0.2479338843	20	0.1652892562
12	35	0.2430555556	25	0.1736111111
13	42	0.2485207101	30	0.1775147929
14	47	0.2397959184	36	0.1836734694
15	56	0.2488888889	42	0.1866666667
16	63	0.24609375	49	0.19140625
17	72	0.2491349481	56	0.1937716263
18	79	0.2438271605	64	0.1975308642
19	90	0.2493074792	72	0.1994459834
20	99	0.2475	81	0.2025

Theorem for Pair of Pants (aAbB)

- For $L \equiv 0 \mod 4$, $\max(\mathrm{SI(L)}) = \frac{L^2}{4} - 1$
- For $L \equiv 1, 3 \mod 4$, $\max(\mathrm{SI(L)}) = \frac{L^2 - 1}{4}$
- For $L \equiv 2 \mod 4$, $\max(\mathrm{SI(L)}) = \frac{L^2}{4} - 2$

Theorem for Torus with One Boundary (abAB)

- For $L \equiv 0, 2 \mod 4$, $\max(\mathrm{SI}(\mathrm{L})) = \left(\frac{L}{2} - 1\right)^2$
- For $L \equiv 1, 3 \mod 4$, $\max(\mathrm{SI}(\mathrm{L})) = \left(\frac{L-3}{2}\right)\left(\frac{L-1}{2}\right)$

More General Theorem

For surfaces aAbB and abAB, the formula for the maximal self-intersection in terms of curve word length is a polynomial of $O(L^2)$ with leading coefficient $\frac{1}{4}$

Future Research

Generalized Conjecture:

For surfaces with boundary, if there exists a relationship between the maximal self-intersection and the curve word length, then this relationship is $O(L^2)$. Also, the leading coefficient of this polynomial approaches

- $\frac{1}{3} = .\overline{33}$ for surfaces with an alphabet of 6 letters
- $\frac{3}{8} = .375$ for surfaces with an alphabet of 8 letters
- $\frac{2}{5} = .4$ for surfaces with an alphabet of 10 letters

Acknowledgements

I would like to thank

- Professor Moira Chas for proposing this project and for working with me every week
- MIT PRIMES-USA for this wonderful opportunity
- and my parents for supporting me throughout