September 18, 2019: David Vogan (MIT), *Centralizers of nilpotent elements, II*

Report on work of Jeffrey Adams and Annegret Paul.

This is a continuation/deepening of last week's talk. Again, the setting is G complex connected reductive algebraic group, $X \in \mathfrak{g}$ a nilpotent element, and

$$\phi: SL(2) \to G, \qquad d\phi(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}) = X.$$

The goal is to describe as completely as possible the (possibly disconnected) reductive group G^ϕ, in terms accessible to explicit computation by the `atlas` software.

Here is a more precise setting. We fix a pinned based root datum for G, corresponding to a maximal torus in a Borel $T \subset B$, and basis vectors X_α for the simple root spaces. We can and do assume that

$$\phi_X: \mathbb{C}^\times \simeq (\text{diagonal matrices in } SL(2)) \to T,$$

and that the corresponding cocharacter $H_X \in X_*(T)$ is dominant.

Now there are two important (computable) Levi subgroups attached to ϕ_X. The first is the *Jacobson-Morozov Levi* $L_{JM} = G^{H_X}$, which is a standard Levi for B, and obviously uniquely determined by H_X. The second is a *Bala-Carter Levi* L_{BC}, which is a probably non-standard Levi containing T, minimal with respect to the requirement that ϕ_X (keeping our fixed H_X can be chosen to map into L_{BC}. This Levi is unique up to conjugation by $W(L_{JM})$. Henceforth we assume that ϕ_X maps to L_{BC}.

Now G^{ϕ_X} is a possibly disconnected reductive group having a maximal torus

$$T_X = \text{identity component of } Z(L_{BC}) \subset T,$$

which is computable, meaning for example that the cocharacter lattice of T_X is a computable sublattice of $X_*(T)$:

$$X_*(T_X) = \{\xi \in X_*(T) \mid \beta(\xi) = 0, \quad \beta \in \Delta(L_{BC}, T)\}.$$

The `atlas` software continues from here to identify the roots and coroots of T_X in $G_0^{\phi_X}$, which is to say the root datum of $G_0^{\phi_X}$.

I'll explain all this (first half of the talk?) In the second half I'll explain what it *means* to specify a disconnected reductive group starting from the root datum of the identity component, and the conjecture of Adams that would make this specification accessible to the software.

Recall that the results here sought can be found in Lusztig's paper "Unipotent almost characters of simple p-adic groups, II," *Transform. Groups* **19** (2014), no. 2, 527–547.