October 3: "On twisted Gelfand pairs through commutativity of a Hecke algebra."

A pair (G, H) of a finite group G and a subgroup H is called a *Gelfand pair* if every irreducible representation of G appears at most once in the representation $\mathbb{C}[G/H]$ of complex-valued functions on G/H. In this case, the Gelfand property is equivalent to the commutativity of the Hecke algebra $\mathbb{C}[H\backslash G/H]$ of bi-H-invariant functions on G.

Given a reductive p-adic group G and a closed subgroup H, one can generalize the Gelfand property to these settings, and a result of Gelfand and Kazhdan gives sufficient conditions for (G, H) to be a Gelfand pair. Unlike the finite situation, here it is not known whether there is a characterization of the Gelfand property through commutativity of an algebra.

In this talk (based on `arXiv:1807.02843`), we define a Hecke algebra for the pair (G, H) as above and show that if the Gelfand-Kazhdan conditions hold then it is commutative. We then explore the connection between commutativity of this algebra and the Gelfand property of (G, H), and show that for spherical pairs, the cuspidal part of this algebra is commutative if and only if the pair (G, H) satisfies the Gelfand property with respect to all irreducible cuspidal representations of G.