October 25: David Vogan (MIT), "Duality for Harish-Chandra modules."

The local Langlands conjecture says approximately that irreducible (complex) representations of a reductive group $G(F)$ over a local field F should be parametrized by homomorphisms of the Galois group $\Gamma(F)$ into the (complex) dual group ${}^\vee G(\mathbb{C})$. In case $F = \mathbb{R}$, this says that representations of $G(\mathbb{R})$ are approximately parametrized by elements y of order two in ${}^\vee G(\mathbb{C})$.

The Cartan classification of real forms says that real forms of complex reductive group G are approximately parametrized by elements x of order two in $G(\mathbb{C})$.

I will explain (following Adams and du Cloux) how to make precise both of these statements, in such a way that certain pairs $(x, y) \in G(\mathbb{C}) \times {}^\vee G(\mathbb{C})$ parametrize representations both for G and for ${}^\vee G$. The resulting correspondence of representations is the duality of the title. It carries "large" representations to "small" ones. A number of rather difficult and subtle properties on one side of the duality become elementary on the other.