November 8: David Vogan (MIT), *Examples of discrete series (after Krötz, Kuit, Opdam, and Schlichtkrull), continued.*

I'll talk about explicit formulas for discrete series for some non-symmetric homogenous spaces G/H. The main technique is to find a larger automorphism group $M \supset G$ of G/H, so that

$$M/L = G/H, \qquad H = L \cap G$$

and M/L is symmetric. Then one gets (at least some) discrete series for G/H as discrete summands of $\pi|_G$ for the (known) discrete series of M/L.

One example is the real hyperboloid

$$U(p,q)/U(p-1,q) = O(2p,2q)/O(2p-1,2q) \qquad p,q \geq 2.$$

The discrete series for $O(2p-1,2q)$ were found by Strichartz in 1972: they are now understood to be cohomologically induced representations

$$\pi_\ell^O = A_{\mathfrak{q}^O}(\chi_\ell), \qquad \ell > -(p+q-1).$$

Here the θ-stable parabolic $\mathfrak{q}^O$ has Levi subgroup

$$L^O = SO(2) \times O(2p-2,q).$$

To understand how these representations restrict to $U(p,q)$, we use a θ-stable parabolic subalgebra

$$\mathfrak{q}^U \subset \mathfrak{u}(p,q)_{\mathbb{C}}, \qquad L^U = U(1) \times U(p-2,q) \times U(1).$$

Using characters ξ_x of $U(1)$, we get cohomologically induced representations

$$\pi_{x,y}^U = A_{\mathfrak{q}^O}(\xi_x \otimes 1 \otimes \xi_{-y}), \qquad x > -(n-1),\ y > -(n-1),\ x+y > -(n-1).$$

Here we write $n = p+q$. Then what appears to be true is

$$\pi_\ell|_{U(p,q)} = \sum_{-(n-1)<x<\ell+(n-1)} \pi_{x,\ell-x}.$$

I'll explain evidence for this, and a proof in many cases.

I hope to discuss parallel computations for

$$G_{2,s}/SU(2,1) = SO(4,3)/SO(4,2), \quad G_{2,s}/SL(3,\mathbb{R}) = SO(4,3)/SO(3,3),$$

$$\mathrm{Spin}(5,4)/\mathrm{Spin}'(4,3) = O(8,8)/O(7,8).$$