October 19: Ivan Cherednik (UNC), "DAHA and unibranch plane curve singularities I and II."

I will present a recent conjecture that connects the geometry of compactified Jacobians of unibranch plane singularities with the DAHA-superpolynomials of algebraic knots. This somewhat resembles the famous Kazhdan-Lusztig conjecture, but (double) Hecke algebras describe here a different kind of geometry, directly related to that of p-adic orbital integrals (in Fundamental Lemma) and theory of affine Springer fibers (the anisotropic case, type A).

The key geometric ingredients of our construction are degrees and dimensions of the generalized Lusztig-Smelt-Piontkowski cells of the flagged Jacobian factors (new objects, to be defined from scratch); there is a connection with the Kazhdan-Lusztig dimension formulas from their 1988 paper, proved later by Bezrukavnikov.

The DAHA-superpolynomials are expected to coincide with the Khovanov-Rozansky polynomials of algebraic knots. They depend on the parameters a, q, t; for instance, $a = -1$, $q = t$ is the case of the Alexander polynomials, which can be directly expressed via the corresponding singularities (no Jacobian factors are necessary).

When $a = 0$, $q = 1$, $t = 1/p$, we conjecture them to coincide with the p-adic orbital integrals; understanding any a, q in the theory of these integrals is a challenge. Our conjecture readily implies that the orbital integrals in type A (!) depend only on the topological (not just analytic) type of singularity, which is another challenge.

Particular instances of our conjecture are due to Gorsky-Mazin ($a = 0$, torus knots); the so-called Cherednik-Danilenko conjecture (any algebraic knots for $a = 0$, $q = 1$). Our conjecture is different from that due to Oblomkov-Rasmussen-Schende; if time permits I will provide some details here via a reformulation of our conjecture in terms of the weight filtration.

This is joint with Ivan Danilenko and Ian Philipp. I may skip some details concerning the DAHA construction, but the geometric superpolynomials will be defined in full (from scratch). Our conjecture is checked by now "beyond a reasonable doubt," including quite involved examples when the Piontkowski cells are not affine.

The first part (at the Lie Groups Seminar) will be mostly about the plane curve singularities and the corresponding geometric superpolynomials. I will briefly discuss the DAHA construction and the relations to affine Springer fibers; plane curve singularities and Jacobian factors will be fully defined (this is an entirely local theory, where the compactified Jacobians are not too involved).

Then (at "Geometric Representation Theory") we will provide details concerning the DAHA construction (any root systems and iterated knots); this is in fact a one-line formula (not much from DAHA theory is really needed). If time permits, its topological invariance will be justified and further relations to orbital integrals and topology will be discussed including the ORS paper and that by them with Gorsky.