

November 16: Leonid Rybnikov (National Research University, Moscow), *Bethe subalgebras and crystals*.

This is a joint project with Joel Kamnitzer. To a semisimple Lie algebra \mathfrak{g} one assigns a monoidal category of \mathfrak{g} -crystals which is, roughly, a combinatorial model for the category of finite-dimensional representations of $U_q(\mathfrak{g})$ in the $q = 0$ limit. This monoidal category is not braided, rather it is a coboundary category where the role of the braid group is played by the cactus group (i.e., fundamental group of the Deligne-Mumford space of real stable rational curves). In particular, there is a natural action of the cactus group on the set of highest elements of any tensor product of \mathfrak{g} -crystals. Bethe subalgebras form a family of commutative subalgebras in the n -th tensor power of $U(\mathfrak{g})$ parametrized by stable rational curves with $n + 1$ marked points. The generators of Bethe subalgebras are known as (higher) Hamiltonians of the Gaudin magnet chain, in particular the quadratic generators of a Bethe subalgebra are the coefficients of the KZ connection. The cactus group acts naturally on the set of eigenlines of a Bethe subalgebra in the space of highest vectors of a tensor product of any n -tuple of finite-dimensional irreducible representations of \mathfrak{g} . We conjecture a natural bijection between this set of eigenlines and the set of highest elements in the tensor product of the corresponding crystals which agrees with the cactus group action. This can be regarded as a $q = 0$ limit of the Drinfeld-Kohno theorem. I will describe this bijection explicitly for $\mathfrak{g} = \mathfrak{sl}_n$.