March 5, 2014: Hiroshi Oda (Takushoku University and MIT), *The abstract radial part formula*

I shall give an exposition of the contents in §§8,9,11–14 of `arXiv:1402.3231`.There are many parallel stories in the representation theories of a real semisimple Lie group $G = NAK$ and the corresponding graded Hecke algebra $\mathbf{H}$. In the last fall I gave a talk on generalization of the radial part formulas for the restriction $C^\infty(G/K) \to C^\infty(A)$ (this roughly corresponds to §§2,4–7 of the same paper.) Put $M = Z_K(A)$ and let $\widehat{K}_M$ be the set of K-types V with non-zero M-fixed part V^M. For each $V \in \widehat{K}_M$ the Weyl group $W = W(G, A)$ naturally acts on V^M and there is a certain natural W-subspace V^M_{single} in V^M. I recently noticed when a $(\mathfrak{g}_\mathbb{C}, K)$-module $\mathcal{M}_G$ and an $\mathbf{H}$-module $\mathcal{M}_\mathbf{H}$ are in the same position of the parallel worlds, there always exists a set of "restriction" maps

$$\widetilde{\Gamma}^V_\mathcal{M} \colon \operatorname{Hom}_K(V, \mathcal{M}_G) \to \operatorname{Hom}_W(V^M_{\text{single}}, \mathcal{M}_\mathbf{H}) \quad (V \in \widehat{K}_M)$$

with some properties that are formally the same with the generalized Chevalley restriction theorem and the generalized radial part formula. So we define a category of such pairs $\mathcal{M} = (\mathcal{M}_G, \mathcal{M}_\mathbf{H})$ and describe some parallel stories in terms of this category:

Poisson transforms,
Knapp-Stein type intertwining operators,
The Helgason-Fourier transform and the Opdam-Cherednik transform.

In each of them two parallel stories are directly linked via $\widetilde{\Gamma}^V_\mathcal{M}$.