December 4, 2013: James Lepowsky (Rutgers), *Logarithmic tensor category theory—an introduction.*

Tensor product operations for modules play a central role in the representation theory of many classical algebraic structures, such as Lie algebras, commutative associative algebras and Hopf algebras, for example. Suitable module categories typically have natural rigid symmetric or braided tensor category structure, and such structure is often very elementary. In vertex operator algebra theory, there is an analogous, natural theory, and it is far more elaborate. The analogue of tensor category structure for modules for a semisimple Lie algebra is "vertex tensor category" structure, developed jointly with Yi-Zhi Huang some years ago. More generally, the analogue for modules for a general Lie algebra is a "logarithmic tensor category theory" developed more recently, jointly with Huang and Lin Zhang. These structures can be "specialized" to braided tensor category structure, retaining the topological information but losing the conformal-geometric information, which includes the nonmeromorphic operator product expansion that had been postulated to exist in rational and, more recently, logarithmic conformal field theory.

This theory amounts to the vertex-algebraic analogue of the routine triviality "Given a Lie algebra $\mathfrak{g}$, consider the symmetric tensor category of $\mathfrak{g}$-modules." In a precise operadic sense, the notion of vertex operator algebra is a natural "complexification" of the notion of Lie algebra, and simultaneously, of the notion of commutative associative algebra; and analogously, the notion of (logarithmic) vertex tensor category is a natural "complexification" of the notion of braided tensor category. Such "complexification" corresponds mathematically to the passage from point particles to strings in conformal field theory and string theory.

I will sketch some key ideas in this work along with some applications and recent developments.