Dirac cohomology, index polynomial and translation functors

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Let G be a connected real reductive Lie group with complexified Lie algebra $\mathfrak{g}$ and let K be a maximal compact subgroup of G. Attached with a $(\mathfrak{g}, K)$-module X is its Dirac cohomology $H(X)$, i.e the module for the spin double cover of K defined as the quotient $\mathrm{Ker}(D_X)/\mathrm{Ker}(D_X) \cap \ \mathrm{Im}(D_X)$ of the kernel of the (algebraic) Dirac operator (associated with X) with the intersection with its image. Dirac cohomology, introduced by Vogan in the late 1990's, plays an important role in the classification of unitary representations of G. In the case when G and K have the same complex rank, the cohomology splits into modules $H^{\pm}(X)$ and defines the Dirac index $I(X)$ of X, as the virtual module $I(X) = H^{+}(X) - H^{-}(X)$. It turns out that this index induces, via Weyl dimension formula, a polynomial which we call the *index polynomial.* After we describe the behavior under Jantzen-Zuckerman translation functors of both Dirac cohomology and Dirac index, we shall discuss relationships between the index polynomial, the character polynomial and the Goldie rank polynomial. This is joint work with Pavle Pandžić (Univ. Zagreb) and David Vogan (MIT).