**November 28, 2012:** In a 1979 paper, Kazhdan and Lusztig introduced the so-called Kazhdan-Lusztig basis of the Hecke algebra associated to a Coxeter group. They posited a number of deep algebraic conjectures about this basis. These conjectures inspired the blossoming field of geometric representation theory, where they were proven for Weyl groups using the intersection cohomology of Schubert varieties, and other sophisticated geometric techniques. The key tool is the vaunted Decomposition Theorem. However, no algebraic proof of these conjectures was discovered, and the conjectures remained open for general Coxeter groups (which have no Schubert varieties).

Soergel proposed an approach to proving these conjectures algebraically. He constructed a category of bimodules, known as Soergel bimodules, which agree with the equivariant intersection cohomology of Schubert varieties in Weyl group type, but are defined in an algebraic and combinatorial way for any Coxeter group. The Soergel conjecture states that the "characters" of the indecomposable Soergel bimodules are precisely the Kazhdan-Lusztig basis. This is an algebraic analog of the decomposition theorem, and it implies Kazhdan and Lusztig's conjectures.

Inspired by de Cataldo and Migliorini's Hodge-theoretic proof of the Decomposition Theorem, we provide an algebraic proof of the Soergel conjecture for a general Coxeter group. Moreover, we show algebraically that Soergel bimodules have the Hodge-theoretic properties expected of an equivariant intersection cohomology space.