September 7: David Vogan (MIT), "Kazhdan-Lusztig polynomials for involutions."

This is mostly a report of work of Lusztig.

Suppose G is a complex reductive algebraic group with Weyl group (W, S), and Hecke algebra $\mathcal{H}$. Suppose that $K \subset G$ is a symmetric subgroup, the group of fixed points of an involutive automorphism θ of G. Write X for the (finite) set of K-equivariant local systems on orbits of K on $\mathcal{B}$, the variety of Borel subgroups of G. Kazhdan and Lusztig thirty years ago introduced polynomials $P_{x,y}$ (for x and y in X) that carry subtle information about representation theory and the singularities of K-orbit closures. All of this is related to a Hecke algebra representation with basis X.

Suppose now that δ is an automorphism of G preserving a pinning and commuting with θ. The group of fixed points W^δ is again a Coxeter group, with generators indexed by the set T of orbits of $\langle\delta\rangle$ on S. There is an unequal parameter Hecke algebra $\widetilde{\mathcal{H}}$ for (W^δ, T).

There is a natural action of δ on X; write Z for the set of fixed points. Lusztig twenty-five years ago (in some cases) and recently (in general) introduced new polynomials $P^\delta_{z,z'}$ (for z and z' in Z) carrying information about representation theory and the singularities of K orbit closures. All of this is related to an $\widetilde{\mathcal{H}}$ representation with basis Z.

A first example is $G = G_1 \times G_1$ with δ the involution interchanging the two factors and $K \simeq G_1$ the diagonal subgroup. In this case $X = W_1$, and Z is the set of involutions (of order one or two) in W_1. The Hecke algebra $\widetilde{H}$ is the Hecke algebra of W_1 with parameter q^2. The representation at $q = 1$ of W_1 is in type A an "involution model," but in other types something new and somewhat different.