November 22: David Vogan (MIT), "Lyusternik-Šnirel'man category for compact Lie groups (after Hunziker-Sepanski)."

Suppose X is a compact manifold. The *Lyusternik-Šnirel'man category of X*, written $\mathrm{cat}(X)$, is the smallest number of contractible open sets required to cover X. Obviously this is equal to 2 if X is a sphere; but not much else is obvious. This number has been studied in part because it is a lower bound for the number of critical points of a smooth function on X.

Singhof proved in 1975 that $\mathrm{cat}(SU(n)) = n$; I will recall the (elementary linear algebra) proof.

Recently Hunziker and Sepanski showed how to generalize Singhof's argument to an arbitrary simple, simply connected compact Lie group G. Such a group has one distinguished conjugacy class $\mathcal{O}_k$ ($0 \leq k \leq \mathrm{rk}(G) = \ell$) for each vertex of its extended Dynkin diagram; the class $\mathcal{O}_0$ is the identity element, and the distinguished classes include all the central elements of G. What Hunziker and Sepanski prove is

$$\mathrm{cat}(G) \leq \sum_{k=0}^{\ell} \mathrm{cat}(\mathcal{O}_k).$$

I'll explain their proof, and their suggestions about how one might hope to calculate the terms on the right.