December 6: George McNinch (Tufts), "Levi factors and linear algebraic groups."

In general, a linear algebraic group over a field k of positive characteristic has no Levi factor—i.e., no complement to its unipotent radical. Moreover, even if a Levi factor exists, in general not all Levi factors are conjugate.

We give a condition for the existence of a Levi factor which is unique up to geometric conjugacy. This condition applies to some linear groups that arise the study of reductive groups over local fields, as follows.

Let K be a "local field" with integers A. If G is a connected and reductive group over K, Bruhat and Tits have associated to G various smooth A-group schemes Q with generic fiber G. These group schemes are called parahoric group schemes; the groups $Q(A)$ of A-sections are known as parahoric subgroups of the group $G(K)$ of K-rational points of G.

Our result implies that if G splits over an unramified extension of K, then the special fiber Q/k—a linear algebraic group over the residue field k of A—has a Levi factor that is unique up to geometric conjugacy.

The existence of a Levi factor for the special fibers Q/k should have application to DeBacker's parametrization of rational nilpotent orbits for G/K.