November 4: Wilfried Schmid (Harvard University), "Hodge structures and unitary representations."

To understand the irreducible unitary representations of a reductive Lie group G, it suffices to consider Harish Chandra modules whose infinitesimal character is real, relative to the weight lattice. As Vogan has pointed out, in this situation the Harish Chandra module carries a hermitian bilinear form which is infinitesimally invariant under U, a compact real form of the complexification of G. It is related to the infinitesimally G-invariant hermitian bilinear form when that exists, especially transparently in the equal rank case. Vogan has used this relationship to formulate a conjecture on the signature character. I shall describe a conjecture to the effect that Saito's theory of mixed Hodge modules can be used to realize the infinitesimally U-invariant hermitian form geometrically, in terms of an infinite dimensional polarized Hodge structure on the Harish-Chandra module. The conjecture would imply Vogan's conjecture on the signature characters, but would also have other consequences. This is joint work with Kari Vilonen.