MULTIPLICITY ONE THEOREMS: THE ARCHIMEDEAN CASE

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Abstract: Let G be one of the classical Lie groups $\mathrm{GL}_n(\mathbb{R})$, $\mathrm{GL}_n(\mathbb{C})$, $\mathrm{O}(p,q)$, $\mathrm{O}_n(\mathbb{C})$, $\mathrm{U}(p,q)$, and let G' be respectively the subgroup $\mathrm{GL}_{n-1}(\mathbb{R})$, $\mathrm{GL}_{n-1}(\mathbb{C})$, $\mathrm{O}(p,q-1)$, $\mathrm{O}_{n-1}(\mathbb{C})$, $\mathrm{U}(p,q-1)$, embedded in G in the standard way. We show that every irreducible Harish-Chandra smooth representation of G' occurs with multiplicity at most one in every irreducible Harish-Chandra smooth representation of G. This is joint work with Binyong Sun of the Chinese Academy of Sciences.

Independently and in a different approach, A. Aizenbud and D. Gourevitch have proved the multiplicity one theorems for the pairs $(\mathrm{GL}_n(\mathbb{R}), \mathrm{GL}_{n-1}(\mathbb{R}))$ and $(\mathrm{GL}_n(\mathbb{C}), \mathrm{GL}_{n-1}(\mathbb{C}))$.