**May 10:** David Vogan (MIT), "Easy spectral gaps (after Jian-shu Li)."

Akshay Venkatesh in his lectures made a great deal of use of "spectral gaps." One setting for spectral gaps is a measure-preserving action of a group $G$ on a probability space $X$. Then $G$ acts by unitary operators on $L^2(X)$, and $L^2$ is the orthogonal direct sum of the constant functions and the space $L^2_0(X)$ of functions having integral equal to zero. To say that $X$ has a spectral gap is to say that the representation of $G$ on $L^2_0(X)$ has no vectors that are "nearly" $G$-invariant. One way to make such a statement precise is to look for an appropriate $G$-invariant (self-adjoint) operator on $L^2(X)$, which acts by zero on $G$-invariant functions, but has spectrum bounded away from zero on $L^2_0(X)$.

In Akshay's examples, the gaps arose from fairly serious arithmetic ideas. But there are interesting cases when gaps arise directly from unitary representation theory: when one can show that no non-trivial unitary representation can be close to the trivial representation. Jian-shu Li has described this situation in some detail for classical groups ("On the first eigenvalue of Laplacian for locally symmetric manifolds"). I will recall his results, and present easy proofs for some of them using Parthasarathy's Dirac operator.