Branching laws for representations of reductive groups.

Suppose G is a real reductive Lie group with maximal compact subgroup K. Suppose $P = MAN$ is a parabolic subgroup of G, δ is a limit-of-discrete series representation of M, and ν is a character of A. The "standard limit representation" attached to these data is

$$I(\delta \otimes \nu) = \mathrm{Ind}_P^G(\delta \otimes \nu \otimes 1).$$

A fundamental problem in representation theory is to describe the restriction to K of each standard limit representation; that is, to write explicit formulas

$$I(\delta \otimes \nu)|_K = \sum_{\mu \in \widehat{K}} m(\mu, \delta)\mu.$$

The non-negative integer $m(\mu, \delta)$ is the multiplicity of the representation μ of K in $I(\delta \otimes \nu)$. There are classical techniques for computing $m(\mu, \delta)$, based on two of the most important special cases: Kostant's formula for the multiplicity of a weight, and the Blattner formula.

The multiplicities $m(\mu, \delta)$ constitute an integer matrix, indexed by $\widehat{K}$ and G-conjugacy classes of pairs (M, δ). I will describe (roughly speaking) an identification of the two index sets and a partial order on them so that the matrix m becomes square and upper triangular with ones on the diagonal. The matrix m is therefore invertible, and the inverse n is also upper triangular with ones on the diagonal. Explicitly, this means that every irreducible representation μ of K can be written as an integer combination of standard limit representations:

$$\mu = \sum_{M,\delta} n(\delta, \mu) I(\delta \otimes \nu)|_K.$$

(This turns out to be a finite formula: the number of terms on the right is bounded, independent of μ.) I will explain how to compute the integers $n(\delta, \mu)$ explicitly. The multiplicity matrix $m(\mu, \delta)$ can then be recovered by easy linear algebra: inverting the matrix n, which is upper triangular with ones on the diagonal.

This work is a part of the Atlas of Lie Groups project, which lives at

`http://atlas.math.umd.edu/`