

**December 7:** Marc van Leeuwen (Poitiers), “Double crystals and Knuth correspondences.” FOLLOWED BY DINNER.

We shall be considering either matrices with entries in  $\{0, 1\}$  (the binary, or Fermionic case) or with entries in  $N$  (the integral, or Bosonic case). One can encode semistandard tableaux of fixed skew shape using such matrices, by recording the weights of the individual rows or columns of the tableau. For a given shape, the condition that determines whether a matrix encodes any tableau of that shape will be called a “tableau condition”. The Littlewood-Richardson rule, or more precisely its generalisation by Zelevinsky that describes the scalar product between two skew Schur functions, can be formulated as the enumeration of matrices, of one of the two types considered, with given row and column sums and that simultaneously satisfy the tableau condition for one of the skew shapes, and a “Littlewood-Richardson” condition for the other skew shape; in terms of matrices this is a very similar condition.

Inspired by this formulation of the Littlewood-Richardson rule, we define “moves” that modify matrices by transferring a unit from one entry to one of its four neighbours. These moves are subject to restrictions that ensure that at most one unit can be moved from a given row or column to a given neighbouring row respectively column. If only horizontal or only vertical moves are considered, the matrices that can be obtained from a given one form a crystal graph of type  $A$ . In addition horizontal and vertical moves commute, whence all the points in a crystal graph for one direction determine isomorphic graphs for the other direction. We call the resulting structure a double crystal.

Both for the integral and the binary case there exist bijections between on one hand the set of matrices with given row and column sums, and on the other hand the set of pairs of semistandard tableaux of equal straight shapes and with weights respectively given by the specified row and column sums (in fact, in the binary case one of the tableaux is transpose semistandard). Such bijections were defined for both cases by Knuth; for the integral case it is known as the RSK correspondence.

Our moves are such that matrices for which either no upward or no leftward moves are possible satisfy a tableau or Littlewood-Richardson condition for some straight shape, and by similarity of the tableau and Littlewood-Richardson conditions, a tableau of straight shape can be associated to such a matrix in each case. One can then obtain a correspondence of “Knuth type” by associating to a matrix  $M$  the tableaux associated to the pair of matrices obtained from  $M$  by exhausting upward respectively leftward moves. For the binary case this is essentially the correspondence defined by Knuth, while for the integral case it is the Burge correspondence, a variation of the RSK correspondence. To obtain the RSK correspondence, one can exhaust upwards and rightward moves. One way to prove this identification of the bijections defined by exhausting moves with known correspondences is to use Fomin’s growth diagrams.