February 16: Shrawan Kumar (UNC), "On the Cachazo-Douglas-Seiberg-Witten conjecture for simple Lie algebras." FOLLOWED BY DINNER.

Let $\mathfrak{g}$ be a finite-dimensional simple Lie algebra over the complex numbers. Consider the exterior algebra $R := \wedge(\mathfrak{g} \oplus \mathfrak{g})$ on two copies of $\mathfrak{g}$. The the algebra R is bigraded with the two copies of $\mathfrak{g}$ sitting in bidegrees $(1,0)$ and $(0,1)$ respectively. To distinguish, we will denote the first copy of $\mathfrak{g}$ by $\mathfrak{g}_1$ and the second copy of $\mathfrak{g}$ by $\mathfrak{g}_2$.

The diagonal adjoint action of $\mathfrak{g}$ gives rise to a $\mathfrak{g}$-algebra structure on R compatible with the bigrading. We isolate three 'standard' copies of the adjoint representation $\mathfrak{g}$ in R^2, where R^2 is the total degree 2 component of R. The $\mathfrak{g}$-module map

$$\partial\colon \mathfrak{g} \to \wedge^2(\mathfrak{g}), \qquad x \mapsto \partial x = \sum_i [x, e_i] \wedge f_i,$$

considered as a map to $\wedge^2(\mathfrak{g}_1)$, will be denoted by c_1, and similarly $c_2\colon \mathfrak{g} \to \wedge^2(\mathfrak{g}_2)$, and $c_3\colon \mathfrak{g} \to \mathfrak{g}_1 \otimes \mathfrak{g}_2)$, $\quad x \mapsto \sum_i [x, e_i] \otimes f_i$, where $e_{i1 \le i \le N}$ is any basis of $\mathfrak{g}$ and $f_{i1 \le i \le N}$ is the dual basis of $\mathfrak{g}$ with respect to the normalized Killing form $\langle , \rangle$ of $\mathfrak{g}$. We denote by C_i the image of c_i.

Let J be the (bigraded) ideal of R generated by the three copies C_1, C_2, C_3 of $\mathfrak{g}$ (in R^2) and define the bigraded $\mathfrak{g}$-algebra

$$A := R/J.$$

The Killing form gives rise to a $\mathfrak{g}$-invariant $S \in A^{1,1}$ given by

$$S := \sum_i e_i \otimes f_i.$$

Motivated by supersymmetric gauge theory, Cachazo-Douglas-Seiberg-Witten made the following conjecture. They proved the conjecture for classical $\mathfrak{g}$. Subsequently Etingof-Kac proved the conjecture for $\mathfrak{g}$ of type G_2 by using the theory of abelian ideals in $\mathfrak{b}$.

Conjecture [CDSW].

(1) *The subalgebra $A^{\mathfrak{g}}$ of $\mathfrak{g}$-invariants in A is generated as an algebra by the element S.*
(2) $S^h = 0$.
(3) $S^{h-1} \neq 0$, *where h is the dual Coxeter number.*

The aim of this talk is to give a uniform proof of the above conjecture part (i). In addition, we give a conjecture, the validity of which would imply part (ii) of the above conjecture.

To prove part (i), we first prove that the graded algebra $B^{\mathfrak{g}}$ is isomorphic with the singular cohomology of a certain finite-dimensional projective subvariety $\mathcal{Y}_2$ of the infinite Grassmannian $\mathcal{Y}$ associated to $\mathfrak{g}$, where $B := R/\langle C_1 \oplus C_2 \rangle$. The definition of the subvariety $\mathcal{Y}_2$ is motivated from the theory of abelian ideals in the Borel subalgebra $\mathfrak{b}$ of $\mathfrak{g}$. This isomorphism is obtained by using Garland's result on the Lie algebra cohomology of $\widehat{\mathfrak{u}} := \mathfrak{g} \otimes t\mathbb{C}[t]$; Kostant's result on the 'diagonal' cohomology of $\widehat{\mathfrak{u}}$ and its connection with abelian ideals in $\mathfrak{b}$; and a certain deformation of the singular cohomology of $\mathcal{Y}$ introduced by Belkale-Kumar.