February 23: Dan Ciubotaru (MIT) "Unitarity of generic spherical parameters and Weyl group representations."

Let G be the $\mathbb{F}$-points of a split reductive algebraic group defined over $\mathbb{F}$ and K a maximal compact subgroup. $\mathbb{F}$ is $\mathbb{R}$ or a p-adic field. The *spherical* irreducible representations of G (those with nontrivial K-fixed vectors) appear as the unique spherical subquotients $L(\nu)$ of principal series $X(\nu)$, ν being an (unramified) character of a maximal split torus A. It is known that one can restrict to the case when ν is real. ν and ν' parametrize the same spherical representation if they are conjugate under the Weyl group W. In conclusion, one parametrizes spherical representations by elements ν in the dominant Weyl chamber.

A spherical representation $L(\nu)$ is Hermitian if and only if $w_0\nu = -\nu$ (w_0 is the long Weyl group element). The unitarity of $L(\nu)$ can be determined then from the signatures of some operators $A_\mu(w_0, \nu)$, where μ is a K-type in the real case and a W-type in the p-adic case. The *generic spherical* representations correspond to the parameters ν such that all $A_\mu(w_0, \nu)$ are nonsingular.

In the p-adic case, these operators were defined by Barbasch-Moy. $A_\mu(w_0, \nu)$ becomes singular (and its signature can change) only on hyperplanes $\langle \nu,\ check\alpha \rangle = 1$ ($\check{\alpha}$ the coroots). Therefore, the spherical generic representations are parametrized by the open regions in the (generalized Catalan) arrangement of hyperplanes. This arrangement has received a lot of attention recently, due to surprising connections with other areas. In the real case, in order to use the same calculation, one needs to restrict to some special (petite) K-types (Barbasch-Vogan).

In this talk, I will briefly review the constructions above and present a simple calculation of the generic spherical parameters. In particular, it is sufficient to check the signature of the operators on two W-types (the reflection and a subrepresentation of its second symmetric power). This fact has an important consequence for the computation of the generic unitary dual in the p-adic case.

The talk is based on joint work with D. Barbasch.