November 5: David Vogan (MIT), "Delorme's proof of Arthur's Paley-Wiener theorem."

The Fourier transform for a unimodular Lie group G carries the convolution algebra $C_c(G)$ of compactly supported continuous functions on G, to the sum over irreducible representations (π, V_π) of $\mathrm{End}(\mathrm{V}_\pi)$. The Plancherel theorem for G describes precisely the image of an extension of this Fourier transform to $L^2(G)$. A Paley-Wiener theorem for G identifies precisely the image of a subalgebra like $C_c^\infty(G)$. Typically the main condition is that in certain complex families π_λ of representations of G, the Fourier transform operators $\pi_\lambda(f)$ should depend holomorphically on λ, with some precise growth estimates.

If G is a reductive group, then these holomorphicity conditions are not enough to characterize the Fourier transform of C_c^∞. There are additional conditions related to the reducibility of some of the representations π_λ. Arthur described these conditions completely twenty years ago (building on the work of many other people). Delorme has given a new proof of Arthur's result, relying less on hard analysis and more on algebraic understanding of representations. I'll outline some of the ingredients of Delorme's argument.