October 15: Pavel Etingof (MIT), "Matrix valued Macdonald functions."

I will tell about a certain special function $F_V(\lambda, \mu)$ attached to a quantum group $U_q(\mathfrak{g})$ and its finite-dimensional representation V. This function depends on two weights λ, μ of the underlying simple Lie algebra $\mathfrak{g}$, and takes values in $\mathrm{End}V[0]$. It is defined as a (renormalized) weighted trace of an intertwining operator for $U_q(\mathfrak{g})$.

If $\mathfrak{g} = sl(n)$ and V has highest weight $mn\omega_1$, then $V[0]$ is one dimensional, and the function F_V (which is scalar valued in this case) is the Macdonald function of type A for parameters $q, t = q^{m+1}$. Macdonald polynomials are obtained by symmetrizing this function. Macdonald functions and polynomials satisfy a bunch of beautiful identities (difference equations, symmetry under $\lambda \leftrightarrow \mu$, inner product, Gaussian integral identities), due to Macdonald, Koornwinder, Mehta, and Cherednik. These identities form the heart of the so called "Macdonald theory".

In the talk I will present a generalization of Macdonald's theory (i.e., the above identities) to the "matrix" case, i.e. to arbitrary $\mathfrak{g}, V$ (obtained jointly with A. Varchenko). From the point of view of integrable systems, this is the theory of quantum spin Ruijsenaars-Shnider models. The main technical tool of this work is the theory of dynamical R-matrices, to which I'll give an introduction.