

COMBINATORICS SEMINAR

Disproving The Neggers Conjecture

John Stembridge
University of Michigan (Visiting MIT)

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Abstract:

The Eulerian polynomial of a poset P is the sum $W(P, t) = \sum t^{d(w)}$, where w ranges over the linear extensions of P and $d(\cdot)$ counts descents. If P is naturally labeled, then $W(P, t)$ is the h -polynomial of the order complex of the lattice of order ideals of P . In 1978, Neggers conjectured that if P is naturally labeled, then $W(P, t)$ should have only real zeros. In turn, this would imply that the Betti numbers of order complexes of distributive lattices are log-concave and unimodal. In 1986, Stanley conjectured that for *every* poset P , $W(P, t)$ should similarly have only real zeros.

Although these conjectures have been the impetus for a wide range of research over the years, in 2004 Petter Brändén discovered an unnaturally labeled counterexample to Stanley's conjecture. More recently, we have found counterexamples to Neggers' original conjecture. In this talk, we will discuss the counterexamples, the structural theory that enabled them to be found, and the implications for future work in the area.